Interference of outgoing electromagnetic waves generated by two point-like sources

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Abstract

An energy-momentum carried by electromagnetic field produced by two point-like charged particles is calculated. Integration region considered in the evaluation of the bound and emitted quantities produced by all points of world lines up to the end points at which particles' trajectories puncture an observation hyperplane $y^0 = t$. Radiative part of the energy-momentum contains, apart from usual integrals of Larmor terms, also the sum of work done by Lorentz forces of point-like charges acting on one another. Therefore, the combination of wave motions (retarded Liénard-Wiechert solutions) leads to the interaction between the sources.

1. Introduction

We consider a closed system of two point electric charges and their electromagnetic field. A charge e_a produces an electromagnetic vector potential A_a^α that satisfies the wave equation

$$\Box A_a^\alpha = -4\pi j_a^\alpha \tag{1.1}$$

together with the Lorentz gauge condition $\partial_\alpha A_a^\alpha = 0$. The vector j_a^α is the charge's current density which is zero everywhere, except at the particle's position it is infinite. For concreteness we imagine that the particles are asymptotically free in the remote past.

The dynamics of electromagnetic field is governed by Maxwell equations with point-like sources. The action of the field of one source on another is described by Lorentz force. The evolution of a-th particle is determinated by the relativistic generalization of Newton's second law where loss of energy due to radiation is taken into account.

The dynamics of the entire system is governed by the action

$$S = \sum_{a=1}^{2}\left(-m_a\int d\tau_a\sqrt{-(\dot z_a)^2} + e_a\int d\tau_a A_{a,\mu}\dot z_a^\mu\right) - \frac{1}{16\pi}\int d^4y f_{\mu\nu}f^{\mu\nu} \tag{1.2}$$

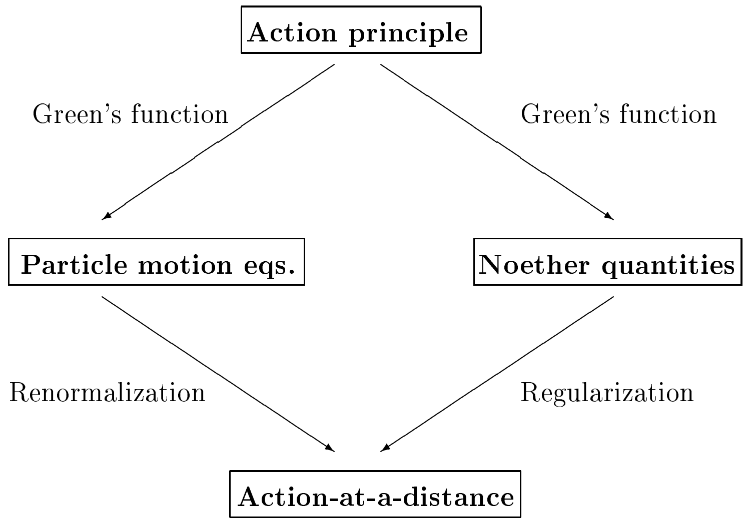

Figure 1: The regularization procedure can be performed in two different ways: (i) one when Green's functions are used in variational equations of motion; (ii) the other when wave solutions are used in Noether conservation laws.

where $f_{\mu\nu} = \sum_a(\partial_\mu A_{a,\nu} - \partial_\nu A_{a,\mu})$. ($a$-th point particle carries electric charge e_a and moves on a world line ζ_a described by functions $z_a^\mu(\tau_a)$, in which τ_a is an evolution parameter; $\dot{z}_a^\mu := dz_a^\mu/d\tau_a$.) Variation on field variables A_a^α yields the Maxwell equations. Liénard-Wiechert fields are the solutions of Maxwell equations with point-like sources.

Since the field $f_{a,\mu\nu} := \partial_\mu A_{a,\nu} - \partial_\nu A_{a,\mu}$ generated by a-th source has a singularity on its world line, demanding that the total action (1.2) be stationary under a variation $\delta z_a^\mu(\tau_a)$ of the world line does not give sensible motion equations. To make sense of the retarded field's action on the particle we should perform the so-called renormalization procedure. It involves manipulation of the divergent self-energy of a point charge. As usual, the infinite Coulomb-like term is linked with the "bare" mass m_a, so that the renormalized mass of particle is considered to be finite.

The principle of least action (1.2) is invariant under ten infinitesimal transformations which constitute the Poincaré group. According to Noether's theorem, these symmetry properties imply conservation laws, i.e. those quantities that do not change with time. In his classical paper [1], Dirac used retarded Liénard-Wiechert solution in the law of conservation of the total four-momentum of a composite (one particle plus field, its own and external) system. It provides the foundation for his derivation of the radiation-reaction force. López and Villarroel [2] substitute the retarded Liénard-Wiechert field in the angular momentum conserved quantity which arises from the invariance of the system under space rotations and Lorentz transformations. The authors arrive at the angular momentum balance equations which is consistent with the Lorentz-Dirac equation.

To find out Noether quantities G^α_{em} carried by electromagnetic field we integrate the Maxwell stress-energy tensor and angular momentum tensor density over a space-like three-surface [3, 4, 5, 6]. We obtain terms of two quite different types: (i) bound, G^α_{bnd},

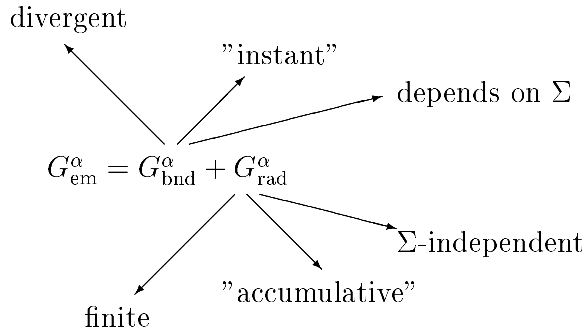

Figure 2: The bound term G^α_{bnd} and the radiative term G^α_{rad} constitute Noether quantity G^α_{em} carried by electromagnetic field. The former diverges while the latter is finite. Bound component depends on instant characteristics of charged particles while the radiative one is accumulated with time. The form of the bound term heavily depends on choosing of an integration surface Σ while the radiative term does not depend on Σ.

which are permanently "attached" to the sources and carried along with them; (ii) radiative, G^α_{rad}, which detach themselves from the charges and lead independent existence (see Fig.2). Within regularization procedure the bound terms are coupled with energy-momentum and angular momentum of "bare" sources, so that already renormalized characteristics G^α_{part} of charged particles are proclaimed to be finite. Noether quantities which are properly conserved become:

$$G^\alpha = G^\alpha_{part} + G^\alpha_{rad}. \tag{1.3}$$

Recently [4] a frontal collision of two asymptotically free charges has been considered. We have calculated how much electromagnetic field momentum and angular momentum flow across hyperplane $\Sigma_t = \{y \in \mathbb{M}_4 : y^0 = t\}$. The crucial issue is that the Maxwell energy-momentum tensor density of entire system

$$4\pi T^{\mu\nu} = f^{\mu\lambda} f^\nu{}_\lambda - 1/4 \eta^{\mu\nu} f^{\kappa\lambda} f_{\kappa\lambda} \tag{1.4}$$

is the sum of individual "one-particle" densities and an "interference" term:

$$T^{\mu\nu} = T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + T^{\mu\nu}_{\rm int}. \tag{1.5}$$

An intrigue feature is that the radiative contribution from the combination of the retarded Liénard-Wiechert fields

$$4\pi T^{\mu\nu}_{\rm int} = f^{\mu\lambda}_{(1)} f^\nu_{(2)\lambda} + f^{\mu\lambda}_{(2)} f^\nu_{(1)\lambda} - 1/4 \eta^{\mu\nu} \left(f^{\kappa\lambda}_{(1)} f^{(2)}_{\kappa\lambda} + f^{\kappa\lambda}_{(2)} f^{(1)}_{\kappa\lambda} \right) \tag{1.6}$$

is then nothing but the sum of work done by Lorentz forces of point-like charges acting on one another. Therefore, an interference of outgoing electromagnetic waves in an *observation hyperplane* Σ_t leads to the interaction between the collided sources. (The differentiation of energy-momentum conserved quantity gives the relativistic generalization

of Newton's second law [5].) This observation gives us an alternative interpretation for the label "int": it stands for "interaction" as well as "interference".

In this paper we study a closed system of two *arbitrarily moving* point-like charges which are asymptotically free in the remote past. The expressions for work done by (retarded) Lorentz forces will be obtained via the rigorous integration of interference parts (1.6) of energy and momentum densities (1.5) over three-dimensional hyperplane Σ_t.

2. Preliminaries

We choose metric tensor $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ for Minkowski space $\mathbb{M}_4$. We use Heaviside-Lorentz system of units with the velocity of light $c=1$. Summation over repeated indices is understood throughout the paper; Greek indices run from 0 to 3, and Latin indices from 1 to 3. The particles' coordinates, velocities etc are labelled a or b.

We consider an arbitrarily moving particles which are asymptotically free in the remote past. Average velocities are not large enough to initiate particle creation and annihilation.

We suppose that the components of momentum four-vector carried by electromagnetic field of particles are [7]

$$p^\nu_{em}(t) = P\int_{\Sigma_t} d\sigma_\mu T^{\mu\nu}\,, \tag{2.1}$$

where $d\sigma_\mu$ is the vectorial surface element on a *observation hyperplane* $\Sigma_t = \{y \in \mathbb{M}_4 : y^0 = t\}$. Particles' world lines

$$\begin{aligned} \zeta_a \;\; : \;\; & \mathbb{R} \to \mathbb{M}_4 \\ & t \mapsto (t, z^i_a(t)) \end{aligned} \tag{2.2}$$

are meant as local sections of trivial bundle $(\mathbb{M}_4, i, \mathbb{R})$ where the projection

$$\begin{aligned} i \;\; : \;\; & \mathbb{M}_4 \to \mathbb{R} \\ & (y^0, y^i) \mapsto y^0 \end{aligned} \tag{2.3}$$

defines the instant form of dynamics [8].

By $T^{\mu\nu}$ we denote the components of the Maxwell energy-momentum density (1.4) where field strengths $f^{\mu\nu}$ are the sum of the retarded Liénard-Wiechert solutions $f^{\mu\nu}_{(1)}$ and $f^{\mu\nu}_{(2)}$ associated with the first and second particles, respectively. So, the total electromagnetic field stress-energy tensor (1.4) becomes the sum (1.5) where the $T^{\mu\nu}_{(a)}$ term is given by the expression (1.4) where "total" field strengths $f^{\mu\nu}$ are replaced by "individual" ones $f^{\mu\nu}_{(a)}$. The interference term (1.6) describes the combination of the *outgoing* electromagnetic waves.

The components $T^{\mu\nu}$ have singularities on particles' trajectories. In equations (2.1) capital letter P denotes the principal value of the singular integral, defined by removing from Σ_t an ε_a-sphere around the a-th particle and then passing to the limit $\varepsilon_a \to 0$.

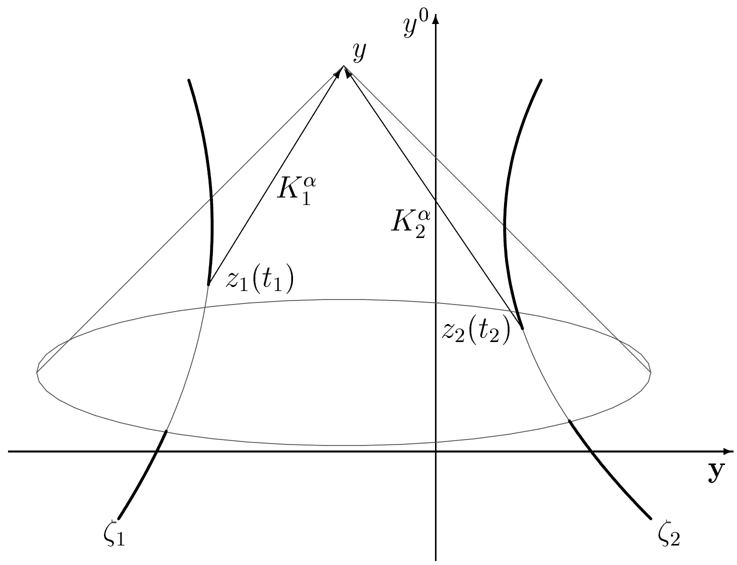

Figure 3: The past light cone with vertex at point $y \in \Sigma_t$ is punctured by the world lines of the 1-st particle and the 2-nd particle at points $z_1(t_1)$ and $z_2(t_2)$, respectively. The vector K_a^α is a null vector pointing from $z_a(t_a) = (t_a, z_a^i(t_a))$ to y.

3. "Interference" coordinate system

The main goal of the present paper is to compute the interference parts of Poincaré group conserved quantities carried by radiation. To perform the volume integration an appropriate coordinate system for flat space-time is necessary.

3.1. Local expressions

The interference terms of energy-momentum and angular momentum at point $y \in \mathbb{M}_4$ depend on the state of the charges' motion at the instants t_1 and t_2 at which their world lines intersect the past light cone (see Fig.3). Coordinates of an observation point y are given by

$$y^\alpha = z_a^\alpha(t_a) + K_a^\alpha \tag{3.1}$$

where K_a^α is the null vector pointing from $z_a(t_a) \in \zeta_a$ to y. Our next task is to find out local expressions for the "light-cone mapping" [9] pictured in Fig.3. We generalise coordinate system presented in [4] where a frontal collision is considered.

The set of curvilinear coordinates contains the "laboratory" time t as well as both the "retarded" times t_1 and t_2. The "laboratory" is a single common parameter defined along all the world lines of the system. To find out local expressions for the components of null-vectors K_1 and K_2 we consider an interference of outgoing electromagnetic waves in hyperplane Σ_t (see Fig.4). By this we mean the intersection of spherical fronts $S_1(O_1, t-t_1)$

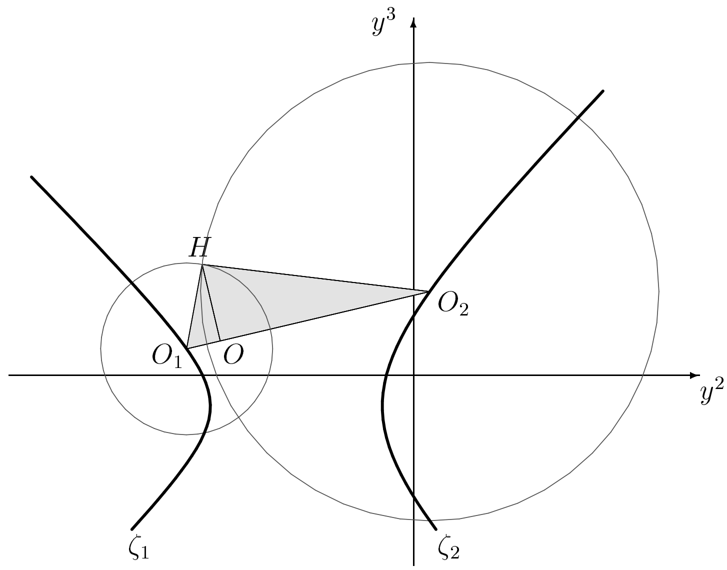

Figure 4: The sphere $S_1(O_1, t - t_1)$ is the intersection of the future light cone with vertex at point $z_1(t_1) \in \zeta_1$ and hyperplane Σ_t. The sphere $S_2(O_2, t - t_2)$ is the intersection of Σ_t and the forward light cone of $z_2(t_2) \in \zeta_2$. Intersection $S_1 \cap S_2$ is the circle $C(O, h)$ with radius $|OH| := h$. It contains an observation point $y \in \Sigma_t$ (see Fig.3).

and $S_2(O_2, t - t_2)$ pictured in Fig.4. It is the circle $C(O, h)$ centred at point

$$\mathbf{Z} = \frac{1}{2}\Big[\mathbf{z}_1(t_1) + \mathbf{z}_2(t_2)\Big] + \frac{(t_1 - t_2)(2t - t_1 - t_2)}{2q^2}\Big[\mathbf{z}_1(t_1) - \mathbf{z}_2(t_2)\Big]. \tag{3.2}$$

Since $|O_1O| = |\mathbf{Z} - \mathbf{z}_1|$ and $|OO_2| = |\mathbf{Z} - \mathbf{z}_2|$, the square of the radius h of the circle can be expressed in the following alternative ways:

$$\begin{aligned} h^2 &= (t - t_1)^2 - |\mathbf{Z} - \mathbf{z}_1|^2 \\ &= (t - t_2)^2 - |\mathbf{Z} - \mathbf{z}_2|^2. \end{aligned} \tag{3.3}$$

The characteristics of the circle are obtained from analysis of the triangle O_1O_2H with sides $|O_1H| = t - t_1$, $|O_2H| = t - t_2$, and $|O_1O_2| = |\mathbf{z}_1(t_1) - \mathbf{z}_2(t_2)| := q$.

To define the coordinates of the points of the circle we translate the origin at the centre (3.2) of the circle $C(O, h)$ and then rotate space axes till new z-axis be directed along three-vector $\mathbf{q} := \mathbf{z}_1 - \mathbf{z}_2$ (see Fig.5). Orthogonal matrix

$$\omega = \begin{pmatrix} \cos\varphi_q & -\sin\varphi_q & 0 \\ \sin\varphi_q & \cos\varphi_q & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\vartheta_q & 0 & \sin\vartheta_q \\ 0 & 1 & 0 \\ -\sin\vartheta_q & 0 & \cos\vartheta_q \end{pmatrix} \tag{3.4}$$

determines the rotation. Finally we obtain coordinate transformation locally written as

$$\begin{aligned} y^0 &= t \\ y^i &= Z^i(t, t_1, t_2) + h(t, t_1, t_2)\omega^i{}_j(t_1, t_2)n^j \end{aligned} \tag{3.5}$$

where $n^j = (\sin\varphi, \cos\varphi, 0)$. Polar angle φ distinguishes the points of circle $C(O,h)$.

To present the local expressions for the *a coordinate system centred on an accelerated world line* of the a-th particle, we rewrite eqs.(3.5) in a manifestly covariant fashion:

$$y^\alpha = z_a^\alpha(t_a) + \Omega^\alpha{}_{\alpha'}(t_1,t_2)k_a^{\alpha'}. \tag{3.6}$$

Four components

$$k_a^0 = t - t_a, \quad k_a^1 = h\sin\varphi, \quad k_a^2 = h\cos\varphi, \quad k_a^3 = (-1)^a|\mathbf{Z} - \mathbf{z}_a| \tag{3.7}$$

satisfy the relations (3.3) and, therefore, constitute null-vector k_a. Having rotated it by orthogonal matrix Ω with components $\Omega_{0\mu} = \Omega_{\mu 0} = \delta_{\mu 0}, \Omega_{ij} = \omega_{ij}$ we obtain the vector K_a pointing from $z_a(t_a) \in \zeta_a$ to $y \in \Sigma_t$ (see Fig.3). The orthogonal matrix ω is given by eq.(3.4); it rotates space axes of the laboratory Lorentz frame (see Fig.5).

Third component of k_a is determined by

$$|\mathbf{Z} - \mathbf{z_a}| = \frac{q}{2} + (-1)^a\frac{(k_2^0)^2 - (k_1^0)^2}{2q}. \tag{3.8}$$

The characteristics $|\mathbf{Z} - \mathbf{z_1}|$ and $|\mathbf{Z} - \mathbf{z_2}|$ are obtained from the analysis of the triangle O_1O_2H with sides $|O_1H| = t - t_1$, $|O_2H| = t - t_2$ and $|O_1O_2| = |\mathbf{z_1}(t_1) - \mathbf{z_2}(t_2)| := q$; they are pictured in Fig.(5).

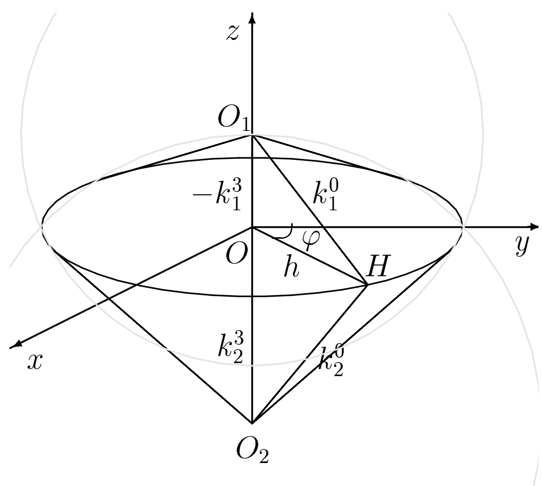

Figure 5: In "momentarily rotating" Lorentz frame $z-$axis is directed along three-vector $\mathbf{q}$. Circle $C(O,h) = S_1 \cap S_2$ lies in Oxy plane; it is centred at the coordinate origin (cf. Fig.4). Polar angle φ distinguishes an observation point $H \in C(O,h)$. Space parts $\mathbf{k}_1$ and $\mathbf{k}_2$ of null vectors k_1 and k_2 are equal to $h\sin\varphi\mathbf{i} + h\cos\varphi\mathbf{j} + k_1^3\mathbf{k}$ and $h\sin\varphi\mathbf{i} + h\cos\varphi\mathbf{j} + k_2^3\mathbf{k}$, respectively.

3.2. Global mapping

To cover the sphere $S_1(z_1(t_1), t - t_1)$ where t_1 is fixed we change the parameter t_2. The starting point is the solution $t_2^{ret}(t_1)$ of algebraic equation

$$t_1 - t_2^{ret} = q(t_1, t_2^{ret}) \tag{3.9}$$

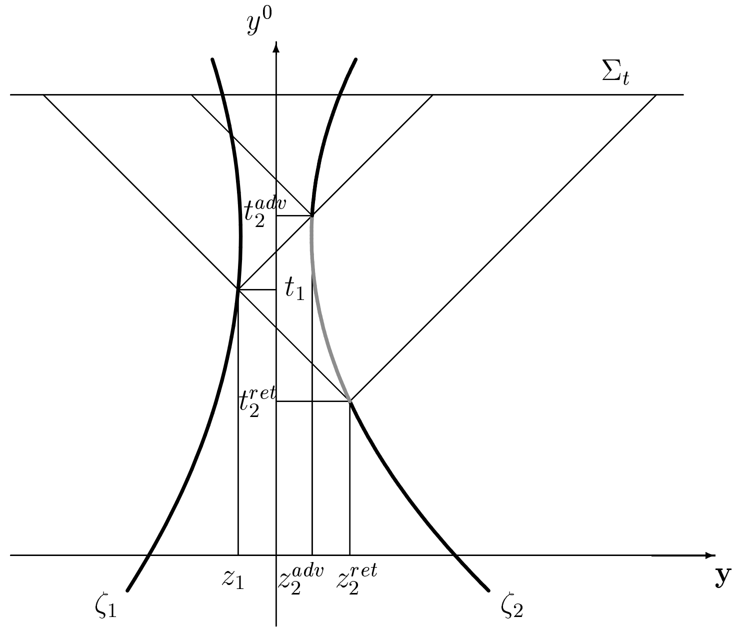

Figure 6: For a given t_1 the retarded time t_2 increases from $t_2^{ret}(t_1)$ to $t_2^{adv}(t_1)$. Minimal value $t_2^{ret}(t_1)$ labels the vertex of forward light cone which is punctured by the world line of the first charge at a given point $(t_1, z_1^i(t_1))$. The world line of the second charge punctures the future light cone of this point at point $(t_2^{adv}(t_1), z_2^i(t_2^{adv}))$.

which describes the future light cone with vertex at $(t_2^{ret}, z_2^i(t_2^{ret}))$ (see Fig.6). The sphere $S_2(z_2^{ret}, t-t_2^{ret})$ touches a given sphere $S_1(z_1(t_1), t-t_1)$ at point N (see Fig.7). If parameter t_2 increases to $t_2^{adv}(t_1)$ being the solution of algebraic equation

$$t_2^{adv} - t_1 = q(t_1, t_2^{adv}) \tag{3.10}$$

the intersection $S_1 \cap S_2^{adv}$ contains the only point S. Equation (3.10) looks as the equation of *backward* light cone of $(t_2^{adv}, z_2^i(t_2^{adv}))$, but it defines the *future* light cone with vertex at $(t_1, z_1^i(t_1))$ (see Fig.6). The sphere S_1 becomes the disjoint union of circles $C(O, h) = S_1 \cap S_2$ if the parameter t_2 changes from $t_2^{ret}(t_1)$ to $t_2^{adv}(t_1)$.

Going along the world line of the first charge we arrive unavoidably at the point $t_1^{ret}(t)$ being the solution of the algebraic equation

$$t - t_1^{ret} = q(t_1^{ret}, t). \tag{3.11}$$

The forward light cone of this point touches the world line of second charge at point $(t, z_2^i(t))$ (see Fig.8). Light cones of upper vertices do not intersect the second world line at all. Spheres $S_1(z_1(t_1), t-t_1)$ determined by $t_1 \in [t_1^{ret}(t), t]$ constitute the region of hyperplane Σ_t which requires another parametrization. For a given instant t_1 from this interval the point S (see Fig.9) is associated with the solution $t_2'(t_1)$ of the following equation:

$$2t - t_1 - t_2' = q(t_1, t_2') \, . \tag{3.12}$$

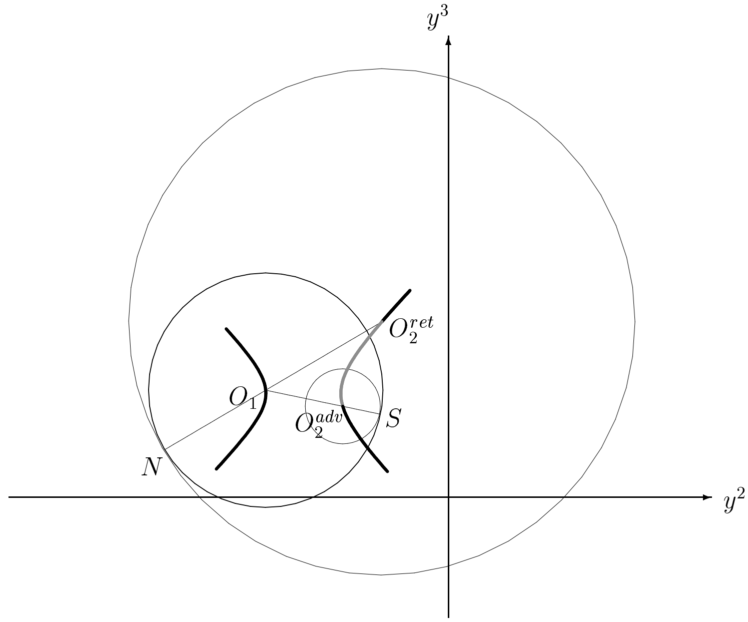

Figure 7: The sphere $S_2(O_2^{ret}, t - t_2^{ret})$ is the intersection of the future light cone at $(t_2^{ret}, z_2^i(t_2^{ret}))$ and Σ_t. It touches a given sphere $S_1(O_1, t - t_1)$ at point N. The sphere $S_2(O_2^{adv}, t - t_2^{adv})$ touches $S_1(z_1, t - t_1)$ at point S. If retarded time t_2 increases from $t_2^{ret}(t_1)$ to $t_2^{adv}(t_1)$ the sphere S_1 is covered by circles $C(O, h) = S_1 \cap S_2$. (A circle $S_1 \cap S_2$ is pictured in Figs.4,5.)

The point N in this figure is still connected with the solution $t_2^{ret}(t_1)$ of equation (3.9).

So, we construct the global coordinate system centred on the world line of the first particle. It bases on the trivial fibre bundle (2.3). A fibre Σ_t is a disjoint union of retarded spheres S_1 centred on the world line of the first particle. A sphere is parametrized by the retarded time of the second particle and the polar angle. Locally the coordinate transformation is given by equations (3.5).

In an analogous way we construct the coordinate system centred on the world line of the second particle. If $t_2 \in]-\infty, t_2^{ret}(t)]$ then $t_1 \in [t_1^{ret}(t_2), t_1^{adv}(t_2)]$; if $t_2 \in [t_2^{ret}(t), t]$ then $t_1 \in [t_1^{ret}(t_2), t_1'(t, t_2)]$, $\varphi \in [0, 2\pi[$. The ends of intervals are defined by the following algebraic equations:

$$
\begin{aligned}
t_2 - t_1^{ret} &= q(t_1^{ret}, t_2) && (3.13)\\
t_1^{adv} - t_2 &= q(t_1^{adv}, t_2) && (3.14)\\
t - t_2^{ret} &= q(t, t_2^{ret}) && (3.15)\\
2t - t_1' - t_2 &= q(t_1', t_2)\,. && (3.16)
\end{aligned}
$$

It is worth noting that the functions $t_1^{ret}(t_2)$ and $t_2^{adv}(t_1)$ are inverted to each other as well as the pair of functions $t_1^{adv}(t_2)$ and $t_2^{ret}(t_1)$ (see Fig.11). For a fixed observation time t the functions $t_1'(t, t_2)$ and $t_2'(t, t_1)$ are inverses too.

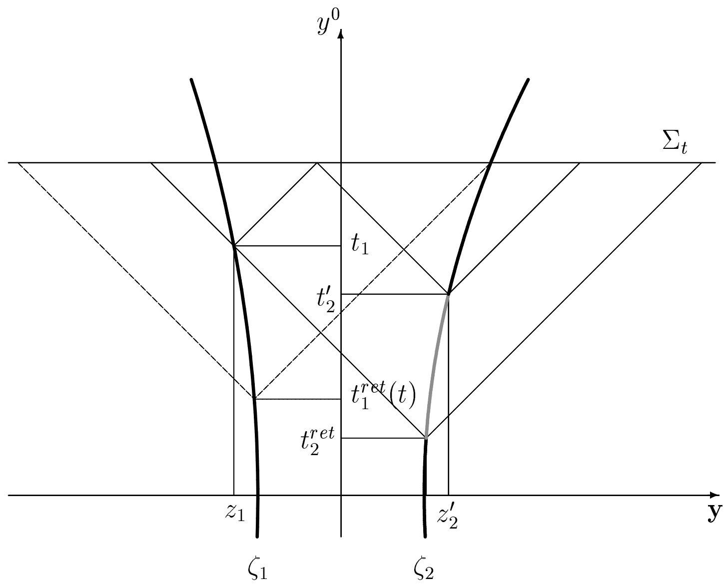

Figure 8: The forward light cone of $(t_1^{ret}(t), z_1^i(t_1^{ret}))$ touches the second world line at the instant of observation. Future light cones of upper vertices do not intersect it at all. For a given $t_1 \in [t_1^{ret}(t), t]$ the parameter t_2 increases from $t_2^{ret}(t_1)$ to $t_2'(t, t_1)$. The maximal value $t_2'(t, t_1)$ labels the vertex of future light cone which touches the forward light cone of $(t_1, z_1^i(t_1))$. The minimal value of t_2 is the solution $t_2^{ret}(t_1)$ of equation (3.9).

4. Electromagnetic fields in terms of "interference" coordinates

Electromagnetic field generated by a−th particle is given by [9, eq.(5.2)]

$$f_{\alpha\beta}^{(a)} = e_a \frac{u_{a,\alpha}\mathsf{k}_{a,\beta} - u_{a,\beta}\mathsf{k}_{a,\alpha}}{(\mathsf{r}_a)^2}\left[1 + \mathsf{r}_a(\mathsf{k}_a \cdot a_a)\right] + e_a \frac{a_{a,\alpha}\mathsf{k}_{a,\beta} - a_{a,\beta}\mathsf{k}_{a,\alpha}}{\mathsf{r}_a}. \tag{4.1}$$

We use sans-serif symbols for the retarded distance [9, 10]

$$\mathsf{r}_a(y) = -\eta_{\alpha\beta}(y^\alpha - z^\alpha(t_a))u^\beta(t_a), \tag{4.2}$$

and for the null vector K_a rescaled by a factor r_a^{-1}:

$$\mathsf{k}_a^\alpha = \frac{1}{\mathsf{r}_a}\left[y^\alpha - z_a^\alpha(t_a)\right]. \tag{4.3}$$

To rewrite expression (4.1) in terms of "interference" curvilinear coordinates consisting of the common evolution parameter t, individual times t_1 and t_2, and angle variable φ, it is advantageous to replace proper time τ_a by evolution parameter t_a. The components of particles' 4-velocities u_a and 4-accelerations a_a, $a = 1, 2$, become [7]

$$u_a^\mu = \gamma_a v_a^\mu(t_a) \tag{4.4}$$
$$a_a^\mu = \gamma_a^4 (v_a \cdot \dot{v}_a) v_a^\mu + \gamma_a^2 \dot{v}_a^\mu \tag{4.5}$$

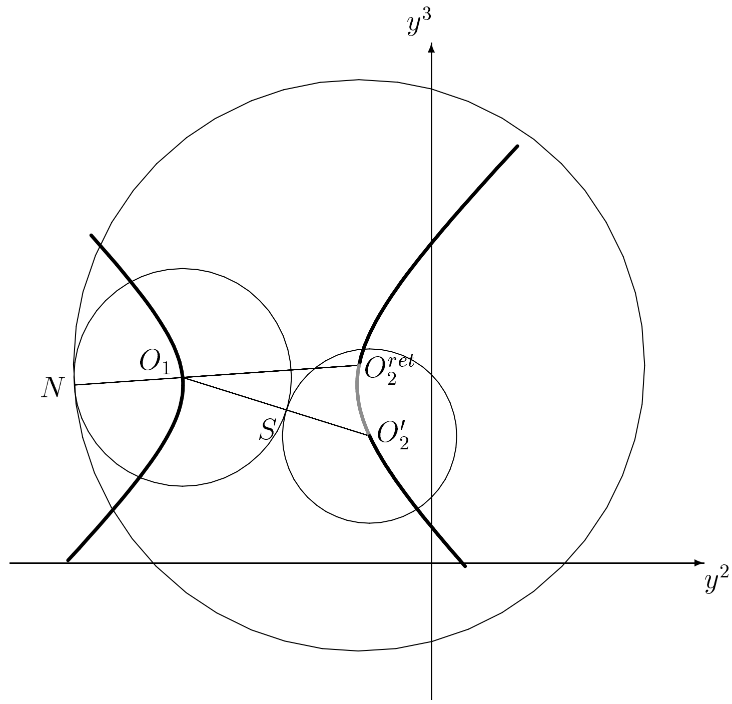

Figure 9: For a given $t_1 \in [t_1^{ret}(t), t]$ the sphere $S_1(O_1, t - t_1)$ is a disjoint union of circles $C(O, h) = S_1 \cap S_2$. Their radius h and centre coordinate Z are determined by t_2. The parameter t_2 increases from $t_2^{ret}(t_1)$ (circle $S_2(O_2^{ret}, t - t_2^{ret})$) to $t_2'(t, t_1)$ (circle $S_2(O_2', t - t_2')$); $\varphi \in [0, 2\pi]$.

where 4-vectors $v_a^\mu = (1, v_a^i(t_a))$, $\dot{v}_a^\mu = (0, \dot{v}_a^i(t_a))$ and factor $\gamma_a := [1 - \mathbf{v}^2]^{-1/2}$. After some algebra, using the relation $\mathsf{k}_a^\mu = K_a^\mu / \mathsf{r}_a$, we obtain

$$
\begin{aligned}
f_{\alpha\beta}^{(a)} &= e_a \frac{v_\alpha(t_a) K_{a,\beta} - v_\beta(t_a) K_{a,\alpha}}{r_a^3} c_a \\
&+ e_a \frac{\dot{v}_\alpha(t_a) K_{a,\beta} - \dot{v}_\beta(t_a) K_{a,\alpha}}{r_a^2}
\end{aligned}
\tag{4.6}
$$

where

$$
c_a = \gamma_a^{-2} + (\mathbf{K}_a \dot{\mathbf{v}}_a), \qquad r_a = K_a^0 - (\mathbf{K}_a \mathbf{v}_a). \tag{4.7}
$$

Having used differential chart (5.2), one can derive the electromagnetic field (4.6) from Liénard-Wiechert potential

$$
A_\alpha^{(a)} = e_a \frac{u_\alpha(t_a)}{\mathsf{r}_a(y)} \tag{4.8}
$$

via the relations $f_{\alpha\beta}^{(a)} = A_{\beta,\alpha}^{(a)} - A_{\alpha,\beta}^{(a)}$.

5. Interference part of electromagnetic field four-momentum

Now, we calculate the interference part of the energy and momentum carried by "two-particle" electromagnetic field:

$$p^\mu_{\rm int}(t) = \int_{\Sigma_t} d\sigma_0 T^{0\mu}_{\rm int}\,. \tag{5.1}$$

An integration hypersurface $\Sigma_t = \{y \in \mathbb{M}_4 : y^0 = t\}$ is a surface of constant t. The surface element is given by $d\sigma_0 = \sqrt{-g}dt_1dt_2d\varphi$ where $\sqrt{-g}$ is the determinant of metric tensor of Minkowski space viewed in curvilinear coordinates (3.5). Differentiation of coordinate transformation (3.5) yields differential chart

$$\begin{pmatrix} \partial/\partial y^0 \\ \partial/\partial y^1 \\ \partial/\partial y^2 \\ \partial/\partial y^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega_{11} & \omega_{12} & \omega_{13} \\ 0 & \omega_{21} & \omega_{22} & \omega_{23} \\ 0 & \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix} \tag{5.2}$$

$$\begin{pmatrix} 1 & \dfrac{t-t_1}{r_1} & \dfrac{t-t_2}{r_2} & 0 \\ 0 & -\dfrac{h\sin\varphi}{r_1} & -\dfrac{h\sin\varphi}{r_2} & \dfrac{\cos\varphi}{h} \\ 0 & -\dfrac{h\cos\varphi}{r_1} & -\dfrac{h\cos\varphi}{r_2} & -\dfrac{\sin\varphi}{h} \\ 0 & \dfrac{|\mathbf{Z}-\mathbf{z_1}|}{r_1} & -\dfrac{|\mathbf{Z}-\mathbf{z_2}|}{r_2} & 0 \end{pmatrix} \begin{pmatrix} \partial/\partial t \\ \partial/\partial t_1 \\ \partial/\partial t_2 \\ \partial/\partial\varphi \end{pmatrix}$$

Its Jacobian gives the determinant of metric tensor mentioned above

$$\sqrt{-g} = \frac{r_1r_2}{q}. \tag{5.3}$$

The volume integration (5.1) can be performed via the coordinate system centred on a world line either of the first particle

$$\left[\int\limits_{-\infty}^{t_1^{ret}(t)} dt_1 \int\limits_{t_2^{ret}(t_1)}^{t_2^{adv}(t_1)} dt_2 + \int\limits_{t_1^{ret}(t)}^{t} dt_1 \int\limits_{t_2^{ret}(t_1)}^{t_2'(t,t_1)} dt_2\right] \int_0^{2\pi} d\varphi \frac{r_1r_2}{q} \tag{5.4}$$

or of the second particle

$$\left[\int\limits_{-\infty}^{t_2^{ret}(t)} dt_2 \int\limits_{t_1^{ret}(t_2)}^{t_1^{adv}(t_2)} dt_1 + \int\limits_{t_2^{ret}(t)}^{t} dt_2 \int\limits_{t_1^{ret}(t_2)}^{t_1'(t,t_2)} dt_1\right] \int_0^{2\pi} d\varphi \frac{r_1r_2}{q}\,. \tag{5.5}$$

The end points of these integrals arise from the interference pictured in Figs.4-9.

5.1. Interference part of zeroth component

In this subsection we trace a series of stages in calculation of the volume integral

$$p_{\rm int}^0 = \int_{\Sigma_t} d\sigma_0 T_{\rm int}^{00}\,. \tag{5.6}$$

In Appendix A we perform the computation in detail.

It is straightforward to substitute the components (4.6) into equation (1.6) to calculate the interference part of electromagnetic field stress-energy tensor. We obtain the following energy density:

$$4\pi T_{\rm int}^{00} = \frac{e_1e_2}{r_1r_2}\left(\frac{\partial^2\Gamma_0}{\partial t_1\partial t_2}\frac{1}{r_1r_2} + \frac{\partial\Gamma_0}{\partial t_1}\frac{c_2}{r_1(r_2)^2} + \frac{\partial\Gamma_0}{\partial t_2}\frac{c_1}{(r_1)^2r_2} + \Gamma_0\frac{c_1c_2}{(r_1)^2(r_2)^2}\right) \tag{5.7}$$

where function

$$\Gamma_0 = \kappa\frac{\partial^2\kappa}{\partial t_1\partial t_2} - \frac{\partial\kappa}{\partial t_1}\frac{\partial\kappa}{\partial t_2}, \qquad \kappa = \frac{1}{2}(k_1^0+k_2^0)^2 - \frac{1}{2}q^2 \tag{5.8}$$

does not depend on angle variable at all.

Taking into account the specific structure of the expression (5.7) which contains the partial derivatives we rewrite the integrand $\sqrt{-g}T_{\bf int}^{00}$ as follows:

$$\begin{aligned}\frac{4\pi}{e_1e_2}\frac{r_1r_2}{q}T_{\rm int}^{00} &= \frac{\partial^2}{\partial t_1\partial t_2}\left(\frac{\Gamma_0}{qr_1r_2}\right) + \frac{\partial}{\partial t_1}\left\{\Gamma_0\left[\frac{c_2}{qr_1(r_2)^2} - \frac{\partial}{\partial t_2}\left(\frac{1}{qr_1r_2}\right)\right]\right\}\\ &+ \frac{\partial}{\partial t_2}\left\{\Gamma_0\left[\frac{c_1}{q(r_1)^2r_2} - \frac{\partial}{\partial t_1}\left(\frac{1}{qr_1r_2}\right)\right]\right\}\\ &+ \Gamma_0\left[\frac{c_1c_2}{q(r_1)^2(r_2)^2} - \frac{\partial}{\partial t_1}\left(\frac{c_2}{qr_1(r_2)^2}\right) - \frac{\partial}{\partial t_2}\left(\frac{c_1}{q(r_1)^2r_2}\right)\right.\\ &+ \left.\frac{\partial^2}{\partial t_1\partial t_2}\left(\frac{1}{qr_1r_2}\right)\right].\end{aligned} \tag{5.9}$$

First of all we should perform the integration over φ (see integration rules (5.4) and (5.5)). The crucial issue is that the integral of the bracketed expression (that which is proportional to Γ_0) over φ vanishes (see Appendix A). Hence the integral of (5.9) over the angle variable has the remarkable properties of being the sum of partial derivatives:

$$\begin{aligned}\int\limits_0^{2\pi} d\varphi\frac{r_1r_2}{q}T_{\rm int}^{00} &= \frac{e_1e_2}{2}\left\{\frac{\partial^2(\Gamma_0\mathcal{D}_0)}{\partial t_1\partial t_2} + \frac{\partial}{\partial t_1}\left[\Gamma_0\left(\mathcal{B}_0 - \frac{\partial\mathcal{D}_0}{\partial t_2}\right)\right]\right.\\ &+ \left.\frac{\partial}{\partial t_2}\left[\Gamma_0\left(\mathcal{C}_0 - \frac{\partial\mathcal{D}_0}{\partial t_1}\right)\right]\right\}.\end{aligned} \tag{5.10}$$

Here

$$\mathcal{D}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi\frac{1}{qr_1r_2}, \quad \mathcal{B}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi\frac{c_2}{qr_1(r_2)^2}, \quad \mathcal{C}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi\frac{c_1}{q(r_1)^2r_2} \tag{5.11}$$

where r_a and c_a are given by eqs.(4.7).

It is natural to integrate the expression being the time derivative with respect to t_1 according to the rule (5.5). The result is

$$\left[\int_{-\infty}^{t_1^{ret}(t)} dt_1 \int_{t_2^{ret}(t_1)}^{t_2^{adv}(t_1)} dt_2 + \int_{t_1^{ret}(t)}^{t} dt_1 \int_{t_2^{ret}(t_1)}^{t_2'(t,t_1)} dt_2\right] \frac{\partial G_2(t_1,t_2)}{\partial t_2} \tag{5.12}$$

$$= \int_{-\infty}^{t_1^{ret}(t)} dt_1 G_2[t_1, t_2^{adv}(t_1)] - \int_{-\infty}^{t} dt_1 G_2[t_1, t_2^{ret}(t_1)] + \int_{t_1^{ret}(t)}^{t} dt_1 G_2[t_1, t_2'(t,t_1)].$$

Having applied the rule (5.4) to the expression of type $\partial G_1/\partial t_1$, we obtain

$$\left[\int_{-\infty}^{t_2^{ret}(t)} dt_2 \int_{t_1^{ret}(t_2)}^{t_1^{adv}(t_2)} dt_1 + \int_{t_2^{ret}(t)}^{t} dt_2 \int_{t_1^{ret}(t_2)}^{t_1'(t,t_2)} dt_1\right] \frac{\partial G_1(t_1,t_2)}{\partial t_1} \tag{5.13}$$

$$= \int_{-\infty}^{t_2^{ret}(t)} dt_2 G_1[t_1^{adv}(t_2), t_2] - \int_{-\infty}^{t} dt_2 G_1[t_1^{ret}(t_2), t_2] + \int_{t_2^{ret}(t)}^{t} dt_2 G_1[t_1'(t,t_2), t_2]$$

The double derivative involved in eq.(5.10) can be written in the form either

$$\frac{\partial}{\partial t_1}\left[\frac{\partial G_0}{\partial t_2}\right] \tag{5.14}$$

or

$$\frac{\partial}{\partial t_2}\left[\frac{\partial G_0}{\partial t_1}\right]. \tag{5.15}$$

Now we choose (5.14) and add this term to $\partial G_1/\partial t_1$.

Therefore, the end points are valuable only in the integration procedure either (5.4) or (5.5). The retarded instant, $t_a^{ret}(t_b)$, and advanced one, $t_a^{ret}(t_b)$, $(a \neq b)$ arise naturally as the limits of integrals. They label the points S and N in which fronts of outgoing electromagnetic waves produced by e_1 and e_2 touch each other (see Figs.7 and 9). Triangle O_1O_2H (see Fig.5) reduces to the line at these moments.

An essential feature of integration is that the functions $t_1^{adv}(t_2)$ and $t_2^{ret}(t_1)$ are inverted to each other (see Fig.11). This cicumstance allows us to change the variables in the "advanced" integral involved in eq.(5.13). Further we couple it with the "retarded" integral of eq.(5.12). We obtain

$$\int_{-\infty}^{t} dt_1 \left[\frac{1-V_1}{1-V_2} G_1 - G_2\right]_{t_2=t_2^{ret}(t_1)} \tag{5.16}$$

where

$$V_a := (\mathbf{n}_q \mathbf{v}_a). \tag{5.17}$$

Scrupulous calculation results the terms of two quite different types: (i) this depends on all previous evolution of the 1-st charge

$$-\int\limits_{-\infty}^{t} dt_1\gamma_1^{-1}F_{21}^0[t_1, t_2^{ret}(t_1)] \tag{5.18}$$

(ii) those determinated by the state of particles' motion at the observation instant only:

$$\begin{aligned} & e_1e_2\left[\frac{1+V_2}{2[t-t_2^{ret}(t_1)](1-V_2)} - \frac{1}{q[t_1, t_2^{ret}(t_1)](1-V_2)}\right]_{t_1\to-\infty}^{t_1=t} \\ = & -\frac{e_1e_2}{2[t-t_2^{ret}(t)]} \end{aligned} \tag{5.19}$$

(see Appendix E, Table 1, left column, first line). The integral (5.18) over world line ζ_1 is then nothing but the zeroth component of the work done by "retarded" Lorentz force acting on the first charge.

It is reasonable that, starting with the retarded Liénard-Wiechert solutions, we obtain the retarded direct field due the 2-nd charge on the 1-st one. A surprising feature is that we can arrive at the expression for the *advanced* direct field within the framework of retarded causality. E.g., one can perform change of variables $(t_1^{ret}(t_2), t_2) \mapsto (t_1, t_2^{adv}(t_1))$ in the *retarded* integral involved in eq.(5.13) and then couple it with the *advanced* expression from eq.(5.12):

$$\int\limits_{-\infty}^{t_1^{ret}(t)} dt_1\left[G_2 - \frac{1+V_1}{1+V_2}G_1\right]^{t_2=t_2^{adv}(t_1)}. \tag{5.20}$$

Having integrated (5.20), we obtain the work done by *advanced* Lorentz force due to the 2-nd charge plus functions of momentary positions of particles:

$$-\int\limits_{-\infty}^{t_1^{ret}(t)} dt_1\gamma_1^{-1}F_{21}^0[t_1, t_2^{adv}(t_1)] + \left[\frac{1}{2k_2^0}\frac{1-V_2}{1+V_2} + \frac{1}{q[1+V_2]}\right]_{t_1\to-\infty}^{t_1\to t_1^{ret}(t)} \tag{5.21}$$

(see Appendix E, Table 1, left column, second line). The matter is that the integral of advanced force due to 2-nd charge over worldline ζ_1 is intimately connected with integral of the retarded force due to 1-st charge over ζ_2:

$$\begin{aligned} & \int\limits_{-\infty}^{t} dt_2\gamma_2^{-1}F_{12}^0[t_1^{ret}(t_2), t_2] - \int\limits_{-\infty}^{t_1^{ret}(t)} dt_1\gamma_1^{-1}F_{21}^0[t_1, t_2^{adv}(t_1)] \\ = & -e_1e_2\left[\frac{-1+(\mathbf{v}_1\cdot\mathbf{v}_2)}{q[1+V_1][1+V_2]} + \frac{1}{q[1+V_1]} + \frac{1}{q[1+V_2]}\right]_{t_2\to-\infty}^{t_2=t} \end{aligned} \tag{5.22}$$

(see Appendix D, eq.(D.11)). Therefore, the advanced expression can be replaced by the retarded one plus functions of momentary positions of particles (see Appendix E, Table 2, left column, second line).

Now we consider the last terms in both eq.(5.13) and eq.(5.12). Since the functions $t'_1(t,t_2)$ and $t'_2(t,t_1)$ are inverses, the sum of these integrals can be written in the form either

$$\int\limits_{t_1^{ret}(t)}^{t} dt_1 \left[G_2 + \frac{1+V_1}{1-V_2}G_1\right]^{t_2=t'_2(t,t_1)} \tag{5.23}$$

or

$$\int\limits_{t_2^{ret}(t)}^{t} dt_2 \left[G_1 + \frac{1-V_2}{1+V_1}G_2\right]^{t_1=t'_1(t,t_2)} . \tag{5.24}$$

Both the expressions result the same function of the end points only:

$$-\left.\frac{1}{2(t-t_2)}\right|_{t_2=t_2^{ret}(t)}^{t_2\to t} = -\lim_{t_2\to t}\frac{e_1e_2}{2(t-t_2)} + \frac{e_1e_2}{2[t-t_2^{ret}(t)]} \tag{5.25}$$

(see Appendix E, Table 1, left column, third line).

Summing up all the contributions (5.18), (5.19), (5.21) where (5.22) is taken into account, and (5.25) we obtain the expression

$$\begin{aligned} p^0_{int} &= -\sum_{b\neq a}\int\limits_{-\infty}^{t} dt_a\gamma_a^{-1}F^0_{ba}(t_a,t_b^{ret}(t_a)) \\ &- \frac{e_1e_2}{q[t_1^{ret}(t),t]}\frac{(\mathbf{v}_2\cdot\mathbf{v}_1)+V_2}{[1+V_1]\,[1+V_2]} - \lim_{t_2\to t}\frac{e_1e_2}{t-t_2}\frac{V_2}{1+V_2} \end{aligned} \tag{5.26}$$

Now we take the double derivative in the form (5.15) and add it to $\partial G_2/\partial t_2$. Analogous calculations give

$$\begin{aligned} p^0_{int} &= -\sum_{b\neq a}\int\limits_{-\infty}^{t} dt_a\gamma_a^{-1}F^0_{ba}(t_a,t_b^{ret}(t_a)) \\ &- \frac{e_1e_2}{q[t,t_2^{ret}(t)]}\frac{(\mathbf{v}_1\cdot\mathbf{v}_2)-V_1}{[1-V_1]\,[1-V_2]} + \lim_{t_1\to t}\frac{e_1e_2}{t-t_1}\frac{V_1}{1-V_1}. \end{aligned} \tag{5.27}$$

(see Appendix E, Table 1, right column).

Having compared eq.(5.26) with (5.27) we are sure that the calculations result the "immovable core" which describes the action of the fields due to one charge on another, and "changeable shell" which expresses the deformation of electromagnetic "clouds" of charged particles due to mutual interaction. Only the immovable terms should be taken into account in the total energy balance equation.

5.2. Interference part of space components

To calculate interference part p^i_{int} of electromagnetic field momentum p^i_{em} we have to integrate the expression

$$4\pi T^{0i}_{int} = f^{0j}_{(1)}f^i_{(2)j} + f^{0j}_{(2)}f^i_{(1)j} \tag{5.28}$$

over three-dimensional hyperplane $y^0 = t$. The electromagnetic field components are given in Section 3.

According to the integration rules (5.4) and (5.5), first of all we perform the angle integration. Then integrand (5.28) looks as follows:

$$\frac{2}{e_1 e_2}\int\limits_0^{2\pi} d\varphi \sqrt{-g}T^{0i}_{int} = \tag{5.29}$$

$$\begin{aligned}
= \quad & \mathcal{A}^i_2 \left[k^0_1 \frac{\partial^2\lambda}{\partial t_1 \partial t_2} + \frac{\partial \lambda}{\partial t_2}\right] + \mathcal{C}^i_2 \left[k^0_1 \frac{\partial^3\lambda}{\partial t_1 \partial t_2^2} + \frac{\partial^2\lambda}{\partial t_2^2}\right] + \mathcal{B}^i_2 k^0_1 \frac{\partial^3\lambda}{\partial t_1^2 \partial t_2} + \mathcal{D}^i_2 k^0_1 \frac{\partial^4\lambda}{\partial t_1^2 \partial t_2^2} \\
+ \quad & \mathcal{A}^i_1 \left[k^0_2 \frac{\partial^2\lambda}{\partial t_1 \partial t_2} + \frac{\partial \lambda}{\partial t_1}\right] + \mathcal{C}^i_1 k^0_2 \frac{\partial^3\lambda}{\partial t_1 \partial t_2^2} + \mathcal{B}^i_1 \left[k^0_2 \frac{\partial^3\lambda}{\partial t_1^2 \partial t_2} + \frac{\partial^2\lambda}{\partial t_1^2}\right] + \mathcal{D}^i_1 k^0_2 \frac{\partial^4\lambda}{\partial t_1^2 \partial t_2^2} \\
+ \quad & \mathcal{A}_0 \left[\lambda(v^i_1 + v^i_2) + k^0_2 v^i_1 \frac{\partial\lambda}{\partial t_2} + k^0_1 v^i_2 \frac{\partial\lambda}{\partial t_1}\right] \\
+ \quad & \mathcal{C}_0 \left[\lambda \dot{v}^i_2 + k^0_2 v^i_1 \frac{\partial^2\lambda}{\partial t_2^2} + k^0_1 \dot{v}^i_2 \frac{\partial\lambda}{\partial t_1}\right] \\
+ \quad & \mathcal{B}_0 \left[\lambda \dot{v}^i_1 + k^0_1 v^i_2 \frac{\partial^2\lambda}{\partial t_1^2} + k^0_2 \dot{v}^i_1 \frac{\partial\lambda}{\partial t_2}\right] \\
+ \quad & \mathcal{D}_0 \left[k^0_2 \dot{v}^i_1 \frac{\partial^2\lambda}{\partial t_2^2} + k^0_1 \dot{v}^i_2 \frac{\partial^2\lambda}{\partial t_1^2}\right]
\end{aligned}$$

where

$$\lambda = 1/2\left[(k^0_1 - k^0_2)^2 - q^2\right]. \tag{5.30}$$

Calligraphic letters $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ denote the following integrals over φ:

$$\begin{aligned}
\mathcal{A}^i_b &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi K^i_b \frac{c_1 c_2}{q(r_1)^2(r_2)^2}, \quad & \mathcal{B}^i_b &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi K^i_b \frac{c_2}{q r_1 (r_2)^2}, \\
\mathcal{C}^i_b &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi K^i_b \frac{c_1}{q(r_1)^2 r_2}, \quad & \mathcal{D}^i_b &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi K^i_b \frac{1}{q r_1 r_2},
\end{aligned} \tag{5.31}$$

where r_a and c_a are given by eqs.(4.7). Functions $\mathcal{B}_0, \mathcal{C}_0$ and $\mathcal{D}_0$ are defined by eqs.(5.11) and function $\mathcal{A}_0$ is

$$\mathcal{A}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \frac{c_1 c_2}{q(r_1)^2(r_2)^2}. \tag{5.32}$$

After some algebra one can rewrite the terms which involve $\mathcal{A}^i_b, \mathcal{B}^i_b, \mathcal{C}^i_b$ and $\mathcal{D}^i_b$ (see the 1-st and the 2-nd lines of eq.(5.29) as follows:

$$\begin{aligned}
& \frac{\partial}{\partial t_1}\left[\Lambda_b\left(\mathcal{B}^i_b - \frac{\partial \mathcal{D}^i_b}{\partial t_2}\right)\right] + \frac{\partial}{\partial t_2}\left[\Lambda_b\left(\mathcal{C}^i_b - \frac{\partial \mathcal{D}^i_b}{\partial t_1}\right)\right] + \frac{\partial^2(\Lambda_b \mathcal{D}^i_b)}{\partial t_1 \partial t_2} \\
+ \quad & \Lambda_b\left(\mathcal{A}^i_b - \frac{\partial \mathcal{B}^i_b}{\partial t_1} - \frac{\partial \mathcal{C}^i_b}{\partial t_2} + \frac{\partial^2 \mathcal{D}^i_b}{\partial t_1 \partial t_2}\right).
\end{aligned} \tag{5.33}$$

Here

$$\Lambda_1 = k_2^0\frac{\partial^2\lambda}{\partial t_1\partial t_2} + \frac{\partial\lambda}{\partial t_1}, \quad \Lambda_2 = k_1^0\frac{\partial^2\lambda}{\partial t_1\partial t_2} + \frac{\partial\lambda}{\partial t_2}. \tag{5.34}$$

Routine scrupulous calculations performed in Appendix B explain that the "non-derivative tails" in eq.(5.33) are proportional to three-velocities:

$$\begin{aligned} \mathcal{A}_1^i - \frac{\partial\mathcal{B}_1^i}{\partial t_1} - \frac{\partial\mathcal{C}_1^i}{\partial t_2} + \frac{\partial^2\mathcal{D}_1^i}{\partial t_1\partial t_2} &= v_1^i(t_1)\left(\mathcal{B}_0 - \frac{\partial\mathcal{D}_0}{\partial t_2}\right) \\ \mathcal{A}_2^i - \frac{\partial\mathcal{B}_2^i}{\partial t_1} - \frac{\partial\mathcal{C}_2^i}{\partial t_2} + \frac{\partial^2\mathcal{D}_2^i}{\partial t_1\partial t_2} &= v_2^i(t_2)\left(\mathcal{C}_0 - \frac{\partial\mathcal{D}_0}{\partial t_1}\right). \end{aligned} \tag{5.35}$$

We add them to the part of integrand (5.29) which involve "zeroth" functions $\mathcal{A}_0, \mathcal{B}_0, \mathcal{C}_0$, and $\mathcal{D}_0$. It is now straightforward (but tedious) matter to rewrite it as the following sum of partial derivatives:

$$\begin{aligned} & \frac{\partial}{\partial t_1}\left[\Lambda_2^i\left(\mathcal{B}_0 - \frac{\partial\mathcal{D}_0}{\partial t_2}\right)\right] + \frac{\partial}{\partial t_2}\left[\Lambda_2^i\left(\mathcal{C}_0 - \frac{\partial\mathcal{D}_0}{\partial t_1}\right)\right] + \frac{\partial^2(\Lambda_2^i\mathcal{D}_0)}{\partial t_1\partial t_2} - \frac{\partial(v_2^i\Lambda_2\mathcal{D}_0)}{\partial t_1} \\ + & \frac{\partial}{\partial t_1}\left[\Lambda_1^i\left(\mathcal{B}_0 - \frac{\partial\mathcal{D}_0}{\partial t_2}\right)\right] + \frac{\partial}{\partial t_2}\left[\Lambda_1^i\left(\mathcal{C}_0 - \frac{\partial\mathcal{D}_0}{\partial t_1}\right)\right] + \frac{\partial^2(\Lambda_1^i\mathcal{D}_0)}{\partial t_1\partial t_2} - \frac{\partial(v_1^i\Lambda_1\mathcal{D}_0)}{\partial t_2} \\ + & \frac{\partial}{\partial t_1}\left[\lambda\left(v_1^i + v_2^i\right)\left(\mathcal{B}_0 - \frac{\partial\mathcal{D}_0}{\partial t_2}\right)\right] + \frac{\partial}{\partial t_2}\left[\lambda\left(v_1^i + v_2^i\right)\left(\mathcal{C}_0 - \frac{\partial\mathcal{D}_0}{\partial t_1}\right)\right] \\ + & \frac{\partial^2}{\partial t_1\partial t_2}\left[\lambda\left(v_1^i + v_2^i\right)\mathcal{D}_0\right] \end{aligned} \tag{5.36}$$

(We keep in mind the identity (A.41)). Recall that Λ_1, Λ_2 are given by eq.(5.34) and

$$\Lambda_1^i = v_1^i k_2^0\frac{\partial\lambda}{\partial t_2}, \quad \Lambda_2^i = v_2^i k_1^0\frac{\partial\lambda}{\partial t_1}. \tag{5.37}$$

Therefore, the integrand (5.29) also becomes the combinations of partial derivatives with respect to time variables, namely the sum of the expressions written in the first line of eq.(5.33) for both $b = 1$ and $b = 2$, and eq.(5.36). Now we apply the integration procedure developed in previous subsection.

Each double derivative involved in (5.29) can be integrated according to the rule either (5.4) or (5.5). There are five terms of this type in this expression. This circumstance implies ten possible ways of integrations. In Appendix E we study two of them in detail (see Table 2 and Table 3).

Firstly we write all the double derivatives in the form (5.14). The integration gives

$$\begin{aligned} p_{int}^i &= -\sum_{b\neq a}\int\limits_{-\infty}^{t} dt_a\gamma_a^{-1}F_{ba}^i(t_a, t_b^{ret}(t_a)) \\ &+ \frac{e_1e_2}{q[t_1^{ret}(t), t]}\frac{\left[-1 + (\mathbf{v}_1\cdot\mathbf{v}_2)\right]n_q^i[t_1^{ret}(t), t]}{[1 + V_1]\,[1 + V_2]} - \frac{e_1e_2}{q[t_1^{ret}(t), t]}\frac{v_1^i[t_1^{ret}(t)]}{1 + V_1} \\ &+ \lim_{t_2\to t}\frac{e_1e_2}{t - t_2}\frac{\left\{n_q^i[t_1^{ret}(t_2), t_2] - v_2^i(t_2)\right\}V_2}{1 - (V_2)^2} \end{aligned} \tag{5.38}$$

Secondly, we express all the mixed derivatives in the form (5.14). We obtain

$$
\begin{aligned}
p_{int}^i &= -\sum_{b\neq a}\int_{-\infty}^{t} dt_a\gamma_a^{-1}F_{ba}^i(t_a,t_b^{ret}(t_a)) \qquad (5.39)\\
&- \frac{e_1e_2}{q[t,t_2^{ret}(t)]}\frac{\left[-1+(\mathbf{v}_1\cdot\mathbf{v}_2)\right]n_q^i[t,t_2^{ret}(t)]}{[1-V_1]\,[1-V_2]} - \frac{e_1e_2}{q[t,t_2^{ret}(t)]}\frac{v_2^i[t_2^{ret}(t)]}{1-V_2}\\
&+ \lim_{t_1\to t}\frac{e_1e_2}{t-t_1}\frac{\left\{n_q^i[t_1,t_2^{ret}(t_1)]+v_1^i(t_1)\right\}V_1}{1-(V_1)^2}
\end{aligned}
$$

Comparing eq.(5.38) with eq.(5.39), we are sure that the form of functions of momentary positions of particles heavily depend on the method of integration. It reinforce our conviction that the changeable "shell" expresses the deformation of electromagnetic "clouds" of "bare" charges due to mutual interaction. Thus only the immovable "core", i.e. sum of work done by Lorentz forces of point-like charges acting on one another, possesses relevant physical sense.

6. Conclusions

Inspection of the energy-momentum carried by the electromagnetic field of two point-like charged particles reveals the essence of renormalization procedure in classical electrodynamics. Volume integration of Maxwell energy-momentum tensor density over three-dimensional hyperplane $y^0 = t$ gives terms of two quite different types: (i) these depend on the state of the particles' motion in the vicinity of the instant of observation t; (ii) those depend on all previous time development of the sources. The former involves diverging quantities while the latter contains finite terms only. Structure of the quantities which are accumulated with time does not depend on choice of integration three-surface while the form of "instant" expression heavily depends on the way of integration.

"Instant" terms are permanently attached to the charges and are carried along with them. By this we mean that a charged particle cannot be separated from its bound electromagnetic "cloud" which has its own 4-momentum [11]. This quantity together with 4-momentum of "bare" charge constitute the *finite* 4-momentum of "dressed" charged particle. (Note that the electromagnetic "clouds" of sources are deformed due to mutual interaction.) All diverging quantities have thus disappeared into the process of *energy-momentum renormalization.*

The terms which are accumulated with time lead to independent existence. They constitute the radiative part of energy-momentum carried by "two-particle" field. It consists of the integrals of individual Larmor relativistic rates over corresponding world lines and the work done by Lorentz forces of point-like charges acting on one another.

The situation considered here, in which the radiation is propagating outward, breaks the time-reversal invariance of Maxwell's theory. Choosing the retarded solution of wave equation (1.1) as the physically-relevant solution, we adopt a specific time direction, when *an interference of outgoing electromagnetic waves* leads to the interaction between

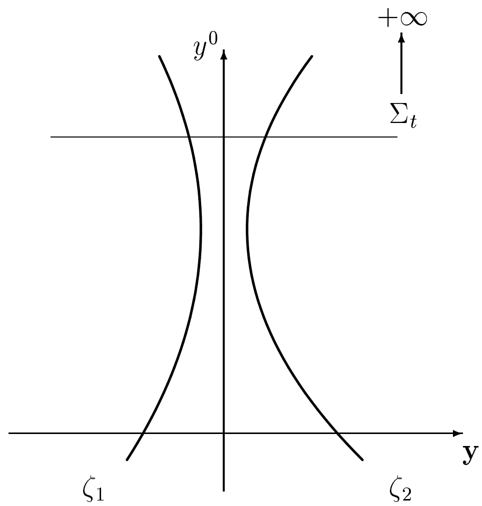

Figure 10: To restore time-reversal invariance we locate the observation hyperplane $y^0 = t$ in the distant future. We suppose that particles are asymptotically free.

the sources. The interference are pictured in a fixed *observation* hyperplane $\Sigma_t = \{y \in \mathbb{M}_4 : y^0 = t\}$. To restore time-reversal invariance we take the limit $t \to +\infty$ and suppose that particles are asymptotically free in the distant future. The relation (D.12) takes the form

$$\int\limits_{-\infty}^{+\infty} dt_a \gamma_a^{-1} F^\mu_{ba}[t_a, t_b^{ret}(t_a)] = \int\limits_{-\infty}^{+\infty} dt_b \gamma_b^{-1} F^\mu_{ab}[t_a^{adv}(t_b), t_b]. \tag{6.1}$$

The work done by retarded Lorentz force of b-th charge over entire world line of a-th one *is equal to* the work done by advanced Lorentz force of a-th particle acting on b-th charge *backward* in time! The sum of "retarded" works involved in the total energy-momentum of our closed (two particles plus field) system

$$\int\limits_{-\infty}^{+\infty} dt_1 \gamma_1^{-1} F^\mu_{21}[t_1, t_2^{ret}(t_1)] + \int\limits_{-\infty}^{+\infty} dt_2 \gamma_2^{-1} F^\mu_{12}[t_1^{ret}(t_2), t_2], \tag{6.2}$$

may be replaced by the linear superposition

$$\begin{aligned} &\frac{1}{2}\left[\int\limits_{-\infty}^{+\infty} dt_1 \gamma_1^{-1} \left(F^\mu_{21}[t_1, t_2^{ret}(t_1)] + F^\mu_{21}[t_1, t_2^{adv}(t_1)]\right)\right. \\ &+ \left.\int\limits_{-\infty}^{+\infty} dt_2 \gamma_2^{-1} \left(F^\mu_{12}[t_1^{ret}(t_2), t_2] + F^\mu_{12}[t_1^{adv}(t_2), t_2]\right)\right] \end{aligned} \tag{6.3}$$

which restore time-reversal invariance. Indeed, the retarded Lorentz force $F^\mu_{ba}[t_a, t_b^{ret}(t_a)]$ becomes the advanced one $F^\mu_{ba}[t_a, t_b^{adv}(t_a)]$ (and vice versa) if the time direction is reversed.

But the retarded causality is still not violated. We consider the interference of *outgoing* waves at distant future instead of a picture in which the radiation is propagated inward.

The situation looks as that described by Wheeler and Feynman [12] where the *absorber theory of radiation* is elaborated. The basic assumption is that the fields which act on a given particle are represented by one-half the retarded plus one-half the advanced Liénard-Wiechert solutions of wave equations. To disappear "incoming" radiation, the authors introduce a *perfect absorber* which cancels the (acausal) advanced part of the fields acting on a given particle and doubles the retarded one.

Our emphasis is on rigorous calculations and exact solutions based on standard electrodynamics. It allows us to substitute the phenomenon of interference of outgoing electromagnetic waves for acausal mechanism of perfect absorbtion in *time-symmetric* action-at-a distance electrodynamics. The interference of outgoing electromagnetic waves (retarded Liénard-Wiechert solutions) ensures the action of the field of one source on another (mutual interaction).

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A. Angle integration of $T^{00}_{\rm int}$

In this Appendix we perform the integration over φ of "double zeroth" component (5.9) of the Maxwell energy-momentum tensor density. Angle-dependent terms involved in energy density have the form

$$\begin{aligned} \mathcal{A}_0 &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \frac{c_1 c_2}{q(r_1)^2(r_2)^2}, \qquad \mathcal{B}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \frac{c_2}{q r_1 (r_2)^2}, \\ \mathcal{C}_0 &= \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \frac{c_1}{q(r_1)^2 r_2}, \qquad \mathcal{D}_0 = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \frac{1}{q r_1 r_2} \end{aligned} \tag{A.1}$$

where the retarded distances are

$$r_a = r_a^0 - r_a^1 \sin\varphi - r_a^2 \cos\varphi. \tag{A.2}$$

The other scalars we use are:

$$c_a = -c_a^0 + c_a^1 \sin\varphi + c_a^2 \cos\varphi \tag{A.3}$$

Here

$$\begin{aligned} r_a^0 &= k_a^0 - (\mathbf{v}_a \cdot \mathbf{n}_q) k_a^3, \quad r_a^1 = \omega_{1j} v_a^j h, \quad r_a^2 = \omega_{2j} v_a^j h & \text{(A.4)} \\ c_a^0 &= -\left(\gamma_a^{-2} + (\dot{\mathbf{v}}_a \cdot \mathbf{n}_q) k_a^3\right), \quad c_a^1 = \omega_{1j} \dot{v}_a^j h, \quad c_a^2 = \omega_{2j} \dot{v}_a^j h. & \text{(A.5)} \end{aligned}$$

It is convenient to introduce three-dimensional manifold $\mathbb{Q}$ with space-favouring metric $g_{\alpha\beta} = \mathrm{diag}(-1,1,1)$. For $\mathbb{Q}$ we put the tangent bundle $T\mathbb{Q}$ being the disjoint union of all tangent spaces $T_x\mathbb{Q}$. A tangent vector with foot point $a \in \mathbb{Q}$ is simply a pair $(a, \mathbf{r})$ with

$$\mathbf{r} = r^\alpha \mathbf{e}_\alpha \in \mathbb{R}^3,$$

where $\mathbf{e}_\alpha := \partial/\partial x^\alpha, \alpha = 0,1,2$, is the standard basis of $\mathbb{R}^3$. We define also cotangent bundle $T^*\mathbb{Q}$ being the disjoint union of $T^*_x\mathbb{Q}$. An one-form with foot point $a \in \mathbb{Q}$ is a pair $(a, \hat{\mathbf{r}})$ with

$$\hat{\mathbf{r}} = r_\beta \hat{\mathbf{e}}^\beta \in \mathbb{R}^3,$$

where $\hat{\mathbf{e}}^\beta, \beta = 0,1,2$, constitute dual basis $\hat{\mathbf{e}}^\beta(\mathbf{e}_\alpha) = \delta^\beta_\alpha$. We shall use $g_{\alpha\beta} = \mathrm{diag}(-1,1,1)$ and its inverse $g^{\alpha\beta} = \mathrm{diag}(-1,1,1)$ to lower and raise indices, respectively.

For each differential manifold one can define the canonical pairing

$$\begin{aligned} \langle , \rangle \quad &: \quad T^*\mathbb{Q} \times_{\mathbb{Q}} T\mathbb{Q} \to \mathbb{R} \\ &\quad\ \langle \hat{\mathbf{r}}_1, \mathbf{r}_2 \rangle \mapsto r_{1,\gamma} r_2^\gamma \end{aligned} \tag{A.6}$$

where both one-form $\hat{\mathbf{r}}_1$ and vector $\mathbf{r}_2$ are of the same foot point. We introduce also the scalar product

$$\begin{aligned} (\cdot) \quad &: \quad T\mathbb{Q} \times_{\mathbb{Q}} T\mathbb{Q} \to \mathbb{R} \\ &\quad\ (\mathbf{r}_1 \cdot \mathbf{r}_2) \mapsto g_{\alpha\beta} r_1^\alpha r_2^\beta \end{aligned} \tag{A.7}$$

which is connected with canonical pairing by the operation of raising indices. And finally, we shall need the norm of vector $\mathbf{r} \in T\mathbb{Q}$

$$\begin{aligned} \|\mathbf{r}\| \quad &= \quad \sqrt{-g_{\alpha\beta} r^\alpha r^\beta} \\ &= \quad \sqrt{(r^0)^2 - (r^1)^2 - (r^2)^2} \end{aligned} \tag{A.8}$$

So, a-th retarded distance r_a becomes the scalar product of the vector with components (A.4) and the null-vector $\mathbf{n}_\varphi := (1, \sin\varphi, \cos\varphi)$ taken with opposite sign.

To go further we express a term of type $(\mathbf{b} \cdot \mathbf{n}_\varphi)/r_1 r_2$ as follows

$$\frac{b}{r_1 r_2} = \frac{-A - C|\mathbf{r}_1|\cos(\varphi + \beta_1)}{r_1^0 - |\mathbf{r}_1|\sin(\varphi + \beta_1)} + \frac{-B + C|\mathbf{r}_2|\cos(\varphi + \beta_2)}{r_2^0 - |\mathbf{r}_2|\sin(\varphi + \beta_2)} \tag{A.9}$$

where scalar b denotes the product $(\mathbf{b} \cdot \mathbf{n}_\varphi)$. The a-th phase β_a is determined by the relations

$$\cos\beta_a = r_a^1/|\mathbf{r}_a|, \quad \sin\beta_a = r_a^2/|\mathbf{r}_a|, \tag{A.10}$$

where

$$\begin{aligned} |\mathbf{r}_a| \quad &= \quad \sqrt{(r_a^1)^2 + (r_a^2)^2} \\ &= \quad h\sqrt{\mathbf{v}_a^2 - (\mathbf{v}_a \cdot \mathbf{n}_q)^2}. \end{aligned} \tag{A.11}$$

The coefficients A, B and C are the solutions of the following system of algebraic equations

$$(C \quad A \quad B) \begin{pmatrix} L_0 & L_1 & L_2 \\ r_{2,0} & r_{2,1} & r_{2,2} \\ r_{1,0} & r_{1,1} & r_{1,2} \end{pmatrix} = (b_0 \quad b_1 \quad b_2) \tag{A.12}$$

where $L_\alpha = g_{\alpha\beta} L^\beta$ and L^β is $^\beta$-th component of the vector

$$L = \begin{pmatrix} \mathbf{e}_0 & \mathbf{e}_1 & \mathbf{e}_2 \\ r_{2,0} & r_{2,1} & r_{2,2} \\ r_{1,0} & r_{1,1} & r_{1,2} \end{pmatrix} \tag{A.13}$$

This can be written more compactly

$$L^\alpha = \varepsilon^{\alpha\beta\gamma} r_{2,\beta} r_{1,\gamma}$$

by use of the Ricci symbol in three dimensions:

$$\varepsilon^{\alpha\beta\gamma} = \begin{cases} +1 \text{ when } \alpha\beta\gamma \text{ is an even permutation of } 0,1,2 \\ -1 \text{ when } \alpha\beta\gamma \text{ is an odd permutation of } 0,1,2 \\ 0 \text{ otherwise} \end{cases} \tag{A.14}$$

In solving the problem (A.9) one is soon led into rather complex expression. Great simplification arise, however, when one uses the binary operation of vector product which is defined as follows:

$$\begin{aligned} [\,\times\,] \quad &: \quad T^*\mathbb{Q} \times_{\mathbb{Q}} T^*\mathbb{Q} \to T\mathbb{Q} \\ & [\hat{\mathbf{r}}_1 \times \hat{\mathbf{r}}_2] \mapsto \varepsilon^{\alpha\beta\gamma} r_{1,\beta} r_{2,\gamma} \end{aligned} \tag{A.15}$$

So, the determinant D of 3×3 matrix in eq.(A.12) becomes the square of vector product of one-forms $\hat{\mathbf{r}}_1$ and $\hat{\mathbf{r}}_2$ given by eqs.(A.4):

$$\begin{aligned} D &= \langle \hat{\mathbf{L}}, \mathbf{L} \rangle = (\mathbf{L})^2 \\ &= [\hat{\mathbf{r}}_2 \times \hat{\mathbf{r}}_1]^2 \\ &= -\langle \hat{\mathbf{r}}_2, \mathbf{r}_2 \rangle \langle \hat{\mathbf{r}}_1, \mathbf{r}_1 \rangle + \langle \hat{\mathbf{r}}_2, \mathbf{r}_1 \rangle^2 \\ &= -(\mathbf{r}_2 \cdot \mathbf{r}_2)(\mathbf{r}_1 \cdot \mathbf{r}_1) + (\mathbf{r}_2 \cdot \mathbf{r}_1)^2 \end{aligned} \tag{A.16}$$

Having solved the system of linear equations (A.12) we obtain

$$\begin{aligned} A &= \frac{(\mathbf{b} \cdot [\hat{\mathbf{r}}_1 \times \hat{\mathbf{L}}])}{D} \\ &= -\frac{1}{D} \Big\{ (\mathbf{b} \cdot \mathbf{r}_2)(\mathbf{r}_1 \cdot \mathbf{r}_1) - (\mathbf{b} \cdot \mathbf{r}_1)(\mathbf{r}_1 \cdot \mathbf{r}_2) \Big\} \end{aligned} \tag{A.17}$$

$$\begin{aligned} B &= -\frac{(\mathbf{b} \cdot [\hat{\mathbf{r}}_2 \times \hat{\mathbf{L}}])}{D} \\ &= \frac{1}{D} \Big\{ (\mathbf{b} \cdot \mathbf{r}_2)(\mathbf{r}_2 \cdot \mathbf{r}_1) - (\mathbf{b} \cdot \mathbf{r}_1)(\mathbf{r}_2 \cdot \mathbf{r}_2) \Big\} \end{aligned} \tag{A.18}$$

$$C = \frac{(\mathbf{b} \cdot \mathbf{L})}{D}. \tag{A.19}$$

The expression of type (A.9) can be integrated over φ via the relations

$$\int_0^{2\pi} d\varphi \frac{1}{1-a\sin\varphi} = \frac{2\pi}{\sqrt{1-a^2}}, \qquad \int_0^{2\pi} d\varphi \frac{\cos\varphi}{1-a\sin\varphi} = 0 \qquad 0 \le a < 1. \tag{A.20}$$

Having integrated (A.9) we obtain

$$\int_0^{2\pi} d\varphi \frac{b}{r_1 r_2} = -2\pi \left(\frac{A}{\|\mathbf{r}_1\|} + \frac{B}{\|\mathbf{r}_2\|} \right). \tag{A.21}$$

Having considered the simplest case of integral $\mathcal{D}_0$ (see eqs.(5.31)), we put the one-form $\hat{\mathbf{b}} = (-1, 0, 0)$. We obtain

$$\mathcal{D}_0 = -\frac{1}{q} \left(\frac{A_0}{\|\mathbf{r}_1\|} + \frac{B_0}{\|\mathbf{r}_2\|} \right), \tag{A.22}$$

where

$$\begin{aligned} A_0 &= \frac{\varepsilon^{0\alpha\beta} r_{1,\alpha} L_\beta}{D} \\ &= -\frac{r_2^0(\mathbf{r}_1\cdot\mathbf{r}_1) - r_1^0(\mathbf{r}_2\cdot\mathbf{r}_1)}{D} \\ B_0 &= -\frac{\varepsilon^{0\alpha\beta} r_{2,\alpha} L_\beta}{D} \\ &= \frac{r_2^0(\mathbf{r}_2\cdot\mathbf{r}_1) - r_1^0(\mathbf{r}_2\cdot\mathbf{r}_2)}{D} \\ C_0 &= \frac{L^0}{D} \end{aligned} \tag{A.23}$$

To calculate $\mathcal{B}_0$ we rewrite the integrand as follows

$$\begin{aligned} \frac{c_2}{r_1(r_2)^2} &= \frac{c_2}{r_1 r_2}\frac{1}{r_2} \\ &= \left(\frac{-A_2 - C_2|\mathbf{r}_1|\cos(\varphi+\beta_1)}{r_1} + \frac{-B_2 + C_2|\mathbf{r}_2|\cos(\varphi+\beta_2)}{r_2} \right) \frac{1}{r_2} \\ &= \frac{-A_2' - C_2'|\mathbf{r}_1|\cos(\varphi+\beta_1)}{r_1^0 - |\mathbf{r}_1|\sin(\varphi+\beta_1)} + \frac{-B_2' + C_2'|\mathbf{r}_2|\cos(\varphi+\beta_2)}{r_2^0 - |\mathbf{r}_2|\sin(\varphi+\beta_2)} \\ &+ \frac{-B_2 + C_2|\mathbf{r}_2|\cos(\varphi+\beta_2)}{[r_2^0 - |\mathbf{r}_2|\sin(\varphi+\beta_2)]^2}, \end{aligned} \tag{A.24}$$

where

$$A_2 = \frac{(\mathbf{c}_2\cdot[\hat{\mathbf{r}}_1\times\hat{\mathbf{L}}])}{D}, \quad B_2 = -\frac{(\mathbf{c}_2\cdot[\hat{\mathbf{r}}_2\times\hat{\mathbf{L}}])}{D}, \quad C_2 = \frac{(\mathbf{c}_2\cdot\mathbf{L})}{D}; \tag{A.25}$$

$$\begin{aligned} A_2' &= -A_2A_0 + C_2C_0\|\mathbf{r}_1\|^2, \quad B_2' = -A_2B_0 + C_2C_0(\mathbf{r}_2\cdot\mathbf{r}_1), \\ C_2' &= -A_2C_0 - C_2A_0 \end{aligned} \tag{A.26}$$

Another relevant integration rules are

$$\int_0^{2\pi} d\varphi \frac{1}{(1-a\sin\varphi)^2} = \frac{2\pi}{(1-a^2)^{3/2}}, \qquad \int_0^{2\pi} d\varphi \frac{\cos\varphi}{(1-a\sin\varphi)^2} = 0 \qquad (A.27)$$
$$0 \le a < 1.$$

Combining these results together with the relations (A.20) for integral of expression (A.24) over φ gives

$$\mathcal{B}_0 = -\frac{1}{q}\left(\frac{A_2'}{\|\mathbf{r}_1\|} + \frac{B_2'}{\|\mathbf{r}_2\|} + \frac{B_2 r_2^0}{\|\mathbf{r}_2\|^3}\right). \qquad (A.28)$$

If one interchanges the indices "first" and "second" in the above expression (A.24), they obtain

$$\mathcal{C}_0 = -\frac{1}{q}\left(\frac{A_1'}{\|\mathbf{r}_1\|} + \frac{B_1'}{\|\mathbf{r}_2\|} + \frac{A_1 r_1^0}{\|\mathbf{r}_1\|^3}\right), \qquad (A.29)$$

where

$$\begin{aligned} A_1 &= \frac{(\mathbf{c}_1 \cdot [\hat{\mathbf{r}}_1 \times \hat{\mathbf{L}}])}{D}, \quad B_1 = -\frac{(\mathbf{c}_1 \cdot [\hat{\mathbf{r}}_2 \times \hat{\mathbf{L}}])}{D}, \quad C_1 = \frac{(\mathbf{c}_1 \cdot \mathbf{L})}{D}; \qquad (A.30) \\ A_1' &= -B_1 A_0 + C_1 C_0 (\mathbf{r}_2 \cdot \mathbf{r}_1), \quad B_1' = -B_1 B_0 + C_1 C_0 \|\mathbf{r}_2\|^2, \\ C_1' &= -B_1 C_0 - C_1 B_0. \qquad (A.31) \end{aligned}$$

We now turn to the calculation of $\mathcal{A}_0$. Transformation of the integrand scales as $(r_1 r_2)^{-2}$ proceeds with the help of eq.(A.9), using identity

$$\frac{c_1 c_2}{(r_1 r_2)^2} = \frac{c_1}{r_1 r_2}\frac{c_2}{r_1 r_2}. \qquad (A.32)$$

The calculation is straightforward, although it involves a fair amount of algebra. Finally we obtain

$$\begin{aligned} \frac{c_1}{r_1 r_2}\frac{c_2}{r_1 r_2} &= \frac{I_1 + I_0 |\mathbf{r}_1| \cos(\varphi + \beta_1)}{(r_1)^2} + \frac{J_1 - J_0 |\mathbf{r}_2| \cos(\varphi + \beta_2)}{(r_2)^2} \qquad (A.33) \\ &+ \frac{-A_{12} - C_{12} |\mathbf{r}_1| \cos(\varphi + \beta_1)}{r_1} + \frac{-B_{12} + C_{12} |\mathbf{r}_2| \cos(\varphi + \beta_2)}{r_2} \end{aligned}$$

where

$$\begin{aligned} I_1 &= A_1 A_2 - C_1 C_2 \|\mathbf{r}_1\|^2, \qquad I_0 = A_1 C_2 + A_2 C_1, \qquad (A.34) \\ J_1 &= B_1 B_2 - C_1 C_2 \|\mathbf{r}_2\|^2, \qquad J_0 = B_1 C_2 + B_2 C_1, \\ A_{12} &= [A_1 B_2 + B_1 A_2 - 2 C_1 C_2 (\mathbf{r}_1 \cdot \mathbf{r}_2)]\, A_0 - J_0 C_0 \|\mathbf{r}_1\|^2 - I_0 C_0 (\mathbf{r}_1 \cdot \mathbf{r}_2), \\ B_{12} &= [A_1 B_2 + B_1 A_2 - 2 C_1 C_2 (\mathbf{r}_1 \cdot \mathbf{r}_2)]\, B_0 - J_0 C_0 (\mathbf{r}_1 \cdot \mathbf{r}_2) - I_0 C_0 \|\mathbf{r}_2\|^2, \\ C_{12} &= [A_1 B_2 + B_1 A_2 - 2 C_1 C_2 (\mathbf{r}_1 \cdot \mathbf{r}_2)]\, C_0 + I_0 B_0 + J_0 A_0. \qquad (A.35) \end{aligned}$$

Using integration rules (A.20) and (A.27), we perform the integration of (A.33) over the angle variable φ:

$$\mathcal{A}_0 = -\frac{1}{q}\left(\frac{A_{12}}{\|\mathbf{r}_1\|} + \frac{B_{12}}{\|\mathbf{r}_2\|} - \frac{I_1 r_1^0}{\|\mathbf{r}_1\|^3} - \frac{J_1 r_2^0}{\|\mathbf{r}_2\|^3}\right). \qquad (A.36)$$

All the coefficients involved in resulting expressions (A.22), (A.28), (A.29), and (A.36) should be rewritten in terms of three-dimensional vectors which denote particles' positions, velocities and accelerations. Substituting components (A.4) and (A.5) into expressions (A.23), (A.25) and (A.30) returns the root coefficients A_i, B_i and $C_i, i = 0, 1, 2$:

$$\begin{aligned} A_0 &= -\frac{([\mathbf{n}_q \times \mathbf{v}_1] \cdot [\mathbf{n}_q \times \mathbf{l}])}{\Delta}, \\ B_0 &= \frac{([\mathbf{n}_q \times \mathbf{v}_2] \cdot [\mathbf{n}_q \times \mathbf{l}])}{\Delta}, \\ C_0 &= \frac{(\mathbf{n}_q \cdot [\mathbf{v}_2 \times \mathbf{v}_1])}{\Delta}, \end{aligned} \tag{A.37}$$

where

$$\Delta = [\mathbf{n}_q \times \mathbf{l}]^2 - h^2(\mathbf{n}_q[\mathbf{v}_2 \times \mathbf{v}_1])^2, \quad \mathbf{l} = r_2^0\mathbf{v}_1 - r_1^0\mathbf{v}_2; \tag{A.38}$$

$$\begin{aligned} A_1 &= \Delta^{-1}\Big\{c_1^0\left([\mathbf{n}_q \times \mathbf{v}_1] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) - r_1^0\left([\mathbf{n}_q \times \dot{\mathbf{v}}_1] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) \\ &+ h^2\left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_1 \times \mathbf{v}_1]\right)\left(\mathbf{n}_q \cdot [\mathbf{v}_1 \times \mathbf{v}_2]\right)\Big\}, \\ B_1 &= -\Delta^{-1}\Big\{c_1^0\left([\mathbf{n}_q \times \mathbf{v}_2] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) - r_2^0\left([\mathbf{n}_q \times \dot{\mathbf{v}}_1] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) \\ &+ h^2\left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_1 \times \mathbf{v}_2]\right)\left(\mathbf{n}_q \cdot [\mathbf{v}_1 \times \mathbf{v}_2]\right)\Big\}, \\ C_1 &= \Delta^{-1}\Big\{- c_1^0\left(\mathbf{n}_q \cdot [\mathbf{v}_2 \times \mathbf{v}_1]\right) + \left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_1 \times \mathbf{l}]\right)\Big\}, \end{aligned} \tag{A.39}$$

$$\begin{aligned} A_2 &= \Delta^{-1}\Big\{c_2^0\left([\mathbf{n}_q \times \mathbf{v}_1] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) - r_1^0\left([\mathbf{n}_q \times \dot{\mathbf{v}}_2] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) \\ &+ h^2\left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_2 \times \mathbf{v}_1]\right)\left(\mathbf{n}_q \cdot [\mathbf{v}_1 \times \mathbf{v}_2]\right)\Big\}, \\ B_2 &= -\Delta^{-1}\Big\{c_2^0\left([\mathbf{n}_q \times \mathbf{v}_2] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) - r_2^0\left([\mathbf{n}_q \times \dot{\mathbf{v}}_2] \cdot [\mathbf{n}_q \times \mathbf{l}]\right) \\ &+ h^2\left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_2 \times \mathbf{v}_2]\right)\left(\mathbf{n}_q \cdot [\mathbf{v}_1 \times \mathbf{v}_2]\right)\Big\}, \\ C_2 &= \Delta^{-1}\Big\{- c_2^0\left(\mathbf{n}_q \cdot [\mathbf{v}_2 \times \mathbf{v}_1]\right) + \left(\mathbf{n}_q \cdot [\dot{\mathbf{v}}_2 \times \mathbf{l}]\right)\Big\}. \end{aligned} \tag{A.40}$$

A complex calculation performed with the help of software system "Maple 8" confirms the key identity

$$\mathcal{A}_0 - \frac{\partial \mathcal{B}_0}{\partial t_1} - \frac{\partial \mathcal{C}_0}{\partial t_2} + \frac{\partial^2 \mathcal{D}_0}{\partial t_1 \partial t_2} = 0. \tag{A.41}$$

It allows us to rewrite the integral of "double zeroth" component of the Maxwell energy-momentum tensor density over φ as the sum (5.10) of partial derivatives in time variables.

It is worth noting that all the coefficients (A.37)-(A.40) and, therefore, expressions (A.22), (A.28), (A.29) and (A.36) depend on h^2, i.e. on the square of the radius of the circle $C(O,h) = S_1 \cap S_2$ (see Figs.4,5). One can express functions $\mathcal{A}_0$, $\mathcal{B}_0$, $\mathcal{C}_0$ and $\mathcal{D}_0$ in form of expansions in powers of h^2. (To simplify the calculations as much as possible we can rewrite the *integrands* of (A.1) as expansions in power h and then *integrate over* φ.) Putting $h^2 \to 0$ we tend to convex neighbourhood of the end points, either S or N (see Figs.7, 9). The identity (A.41) is also valid in the immediate vicinity of the end points. (Differentiating functions $\mathcal{B}_0$, $\mathcal{C}_0$ and $\mathcal{D}_0$ in time variables we must keep in mind that $\partial h^2/\partial t_a$ does not vanish even if $h^2 \to 0$.)

B. Angle integration of T^{0i}_{int}

To express the integral of T^{0i}_{int} over φ as a combination of partial derivatives in time variables we have to calculate the following "tails":

$$\mathcal{A}^i_b - \frac{\partial \mathcal{B}^i_b}{\partial t_1} - \frac{\partial \mathcal{C}^i_b}{\partial t_2} + \frac{\partial^2 \mathcal{D}^i_b}{\partial t_1 \partial t_2} \tag{B.1}$$

(see eq.(5.33). By calligraphic letters we denote the integrals over angle variable:

$$\begin{aligned}
\mathcal{A}^i_b &= \frac{1}{2\pi}\int_0^{2\pi} d\varphi K^i_b \frac{c_1 c_2}{q(r_1)^2(r_2)^2}, \qquad \mathcal{B}^i_b = \frac{1}{2\pi}\int_0^{2\pi} d\varphi K^i_b \frac{c_2}{q r_1 (r_2)^2}, \\
\mathcal{C}^i_b &= \frac{1}{2\pi}\int_0^{2\pi} d\varphi K^i_b \frac{c_1}{q(r_1)^2 r_2}, \qquad \mathcal{D}^i_b = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \frac{K^i_b}{q r_1 r_2}
\end{aligned} \tag{B.2}$$

where

$$K^i_b = k^3_b n^i_q + h\omega^i_1 \sin\varphi + h\omega^i_2 \cos\varphi. \tag{B.3}$$

The integration can be handled via the relation (A.9). The simplest term $\mathcal{D}^i_b$ becomes:

$$\mathcal{D}^i_b = -\frac{1}{q}\left(\frac{A^i_b}{\|\mathbf{r}_1\|} + \frac{B^i_b}{\|\mathbf{r}_2\|}\right). \tag{B.4}$$

Here

$$\begin{aligned}
A^i_b = k^3_b n^i_q A_0 \; &- \; \frac{1}{\Delta}\Big\{ \left[v^i_2 - n^i_q(\mathbf{n}_q \mathbf{v}_2)\right](\mathbf{r}_1 \cdot \mathbf{r}_1) \\
&- \left[v^i_1 - n^i_q(\mathbf{n}_q \mathbf{v}_1)\right](\mathbf{r}_1 \cdot \mathbf{r}_2)\Big\}, \\
B^i_b = k^3_b n^i_q B_0 \; &+ \; \frac{1}{\Delta}\Big\{ \left[v^i_2 - n^i_q(\mathbf{n}_q \mathbf{v}_2)\right](\mathbf{r}_2 \cdot \mathbf{r}_1) \\
&- \left[v^i_1 - n^i_q(\mathbf{n}_q \mathbf{v}_1)\right](\mathbf{r}_2 \cdot \mathbf{r}_2)\Big\}
\end{aligned} \tag{B.5}$$

where A_0, B_0 and Δ are defined by eqs.(A.37) and (A.38). But we find out the expressions (B.1) in another way.

To simplify the calculations as much as possible we express the integrands of eqs.(B.2) in form of expansions in powers of h. Thanks to exponential operator

$$Y := \exp\left[-\sum_a h v_a^j(\omega_{j1}\sin\varphi + \omega_{j2}\cos\varphi)\frac{d}{dr_a^0}\right] \tag{B.6}$$

we remove harmonic functions from denominators and then integrate over φ. In fact, we deal with the flow of the vector field in between the square brackets of eq.(B.6). It maps an open neighbourhood of end points either S or N to an open vicinity of another point of integral curve of this vector field [13]. It is sufficient to compute "tails" (B.1) at the end points where $h^2 = 0$ (see Figs.7 and 9).

At these end points the term $\mathcal{A}_b^i$ is as follows:

$$\begin{aligned}
\mathcal{A}_b^i &= \int_0^{2\pi} d\varphi\ K_b^i \frac{c_1 c_2}{q(r_1)^2(r_2)^2}\bigg|_{h^2=0} \\
&= k_b^3 n_q^i \frac{c_1^0 c_2^0}{q(r_1^0)^2(r_2^0)^2} \\
&= k_b^3 n_q^i \mathcal{A}_0\big|_{h^2=0}.
\end{aligned} \tag{B.7}$$

Since derivatives $\partial h^2/\partial t_a, a = 1, 2$, do not vanish whenever $h^2 = 0$, we should expand $\mathcal{C}_b^i$ and $\mathcal{B}_b^i$ up to the first order of this small parameter:

$$\begin{aligned}
\mathcal{B}_b^i &= k_b^3 n_q^i \mathcal{B}_0 \\
&+ \frac{h^2}{2qr_1^0(r_2^0)^2}\left[-c_2^0\left(2\frac{v_2^i - n_q^i(\mathbf{n}_q\mathbf{v}_2)}{r_2^0} + \frac{v_1^i - n_q^i(\mathbf{n}_q\mathbf{v}_1)}{r_1^0}\right) + \dot{v}_2^i - n_q^i(\mathbf{n}_q\dot{\mathbf{v}}_2)\right] \\
&+ O(h^2)
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\mathcal{C}_b^i &= k_b^3 n_q^i \mathcal{C}_0 \\
&+ \frac{h^2}{2q(r_1^0)^2 r_2^0}\left[-c_1^0\left(\frac{v_2^i - n_q^i(\mathbf{n}_q\mathbf{v}_2)}{r_2^0} + 2\frac{v_1^i - n_q^i(\mathbf{n}_q\mathbf{v}_1)}{r_1^0}\right) + \dot{v}_1^i - n_q^i(\mathbf{n}_q\dot{\mathbf{v}}_1)\right] \\
&+ O(h^2)
\end{aligned} \tag{B.9}$$

Symbols $\mathcal{B}_0$ and $\mathcal{C}_0$ denote the expansions of corresponding integrals (5.31) in powers of h^2:

$$\begin{aligned}
\mathcal{B}_0 &= \frac{-c_2^0}{qr_1^0(r_2^0)^2} \\
&+ \frac{-c_2^0}{2qr_1^0(r_2^0)^2}\left[3\frac{[\mathbf{n}_q\mathbf{v}_2]^2}{(r_2^0)^2} + 2\frac{([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])}{r_1^0 r_2^0} + \frac{[\mathbf{n}_q\mathbf{v}_1]^2}{(r_1^0)^2}\right] h^2 \\
&+ \frac{1}{2qr_1^0(r_2^0)^2}\left[2\frac{([\mathbf{n}_q\dot{\mathbf{v}}_2][\mathbf{n}_q\mathbf{v}_2])}{r_2^0} + \frac{([\mathbf{n}_q\dot{\mathbf{v}}_2][\mathbf{n}_q\mathbf{v}_1])}{r_1^0}\right] h^2 + O(h^2)
\end{aligned} \tag{B.10}$$

$$\mathcal{C}_0 = \frac{-c_1^0}{q(r_1^0)^2 r_2^0} \tag{B.11}$$

$$
\begin{aligned}
&+ \quad \frac{-c_1^0}{2q(r_1^0)^2r_2^0}\left[\frac{[\mathbf{n}_q\mathbf{v}_2]^2}{(r_2^0)^2}+2\frac{([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])}{r_1^0r_2^0}+3\frac{[\mathbf{n}_q\mathbf{v}_1]^2}{(r_1^0)^2}\right]h^2\\
&+ \quad \frac{1}{2q(r_1^0)^2r_2^0}\left[\frac{([\mathbf{n}_q\dot{\mathbf{v}}_1][\mathbf{n}_q\mathbf{v}_2])}{r_2^0}+2\frac{([\mathbf{n}_q\dot{\mathbf{v}}_1][\mathbf{n}_q\mathbf{v}_1])}{r_1^0}\right]h^2+O(h^2)
\end{aligned}
$$

The last expansion we shall need is

$$
\begin{aligned}
\mathcal{D}_b^i &= k_b^3 n_q^i \mathcal{D}_0 \\
&+ \frac{h^2}{2qr_1^0r_2^0}\left[\frac{v_2^i-n_q^i(\mathbf{n}_q\mathbf{v}_2)}{r_2^0}+\frac{v_1^i-n_q^i(\mathbf{n}_q\mathbf{v}_1)}{r_1^0}\right]\\
&+ \frac{h^4}{8qr_1^0r_2^0}\left\{3\frac{[v_2^i-n_q^i(\mathbf{n}_q\mathbf{v}_2)][\mathbf{n}_q\mathbf{v}_2]^2}{(r_2^0)^3}\right.\\
&+ \frac{2[v_2^i-n_q^i(\mathbf{n}_q\mathbf{v}_2)]([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])+[v_1^i-n_q^i(\mathbf{n}_q\mathbf{v}_1)][\mathbf{n}_q\mathbf{v}_2]^2}{r_1^0(r_2^0)^2}\\
&+ \frac{2[v_1^i-n_q^i(\mathbf{n}_q\mathbf{v}_1)]([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])+[v_2^i-n_q^i(\mathbf{n}_q\mathbf{v}_2)][\mathbf{n}_q\mathbf{v}_1]^2}{(r_1^0)^2r_2^0}\\
&+ \left. 3\frac{[v_1^i-n_q^i(\mathbf{n}_q\mathbf{v}_1)][\mathbf{n}_q\mathbf{v}_1]^2}{(r_1^0)^3}\right\}+O(h^4).
\end{aligned}
\tag{B.12}
$$

By $\mathcal{D}_0$ we denote the following expansion:

$$
\begin{aligned}
\mathcal{D}_0 &= \frac{1}{qr_1^0r_2^0}\\
&+ \frac{h^2}{2qr_1^0r_2^0}\left\{\frac{[\mathbf{n}_q\mathbf{v}_1]^2}{(r_1^0)^2}+\frac{([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])}{r_1^0r_2^0}+\frac{[\mathbf{n}_q\mathbf{v}_2]^2}{(r_2^0)^2}\right\}\\
&+ \frac{h^4}{8qr_1^0r_2^0}\left\{3\frac{[\mathbf{n}_q\mathbf{v}_1]^4}{(r_1^0)^4}+3\frac{[\mathbf{n}_q\mathbf{v}_1]^2([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])}{(r_1^0)^3r_2^0}\right.\\
&+ \frac{2([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])^2+[\mathbf{n}_q\mathbf{v}_1]^2[\mathbf{n}_q\mathbf{v}_2]^2}{(r_1^0)^2(r_2^0)^2}\\
&+ \left. 3\frac{[\mathbf{n}_q\mathbf{v}_2]^2([\mathbf{n}_q\mathbf{v}_1][\mathbf{n}_q\mathbf{v}_2])}{r_1^0(r_2^0)^3}+3\frac{[\mathbf{n}_q\mathbf{v}_2]^4}{(r_2^0)^4}\right\}+O(h^4).
\end{aligned}
\tag{B.13}
$$

Our final task will be to compute expression (B.1). When we differentiate functions $\mathcal{B}_b^i$, $\mathcal{C}_b^i$ and $\mathcal{D}_b^i$ we must keep in mind that derivatives of h^2 with respect to t_a do not vanish even if $h^2 \to 0$. With a degree of accuracy sufficient for our purposes we obtain

$$
\begin{aligned}
\mathcal{A}_1^i-\frac{\partial\mathcal{B}_1^i}{\partial t_1}-\frac{\partial\mathcal{C}_1^i}{\partial t_2}+\frac{\partial^2\mathcal{D}_1^i}{\partial t_1\partial t_2} &= v_1^i\left(\mathcal{B}_0-\frac{\partial\mathcal{D}_0}{\partial t_2}\right)\\
\mathcal{A}_2^i-\frac{\partial\mathcal{B}_2^i}{\partial t_1}-\frac{\partial\mathcal{C}_2^i}{\partial t_2}+\frac{\partial^2\mathcal{D}_2^i}{\partial t_1\partial t_2} &= v_2^i\left(\mathcal{C}_0-\frac{\partial\mathcal{D}_0}{\partial t_1}\right)
\end{aligned}
\tag{B.14}
$$

Substituting these relations into eqs.(5.33) returns the integral (5.29) of interference part of the momentum density T^{0i}_{int} over φ as the combination of partial derivatives in time variables.

C. Direct particle fields and Lorenz forces

In classical electrodynamics the four-dimensional delta function of the square of the interval between points A and B is Green's function of the wave operator. The delta function ensures that the typical points A and B on the worldlines of point-like charges a and b interact if and only if they are connectible by a null ray. The interaction is described by Lorentz force, i.e. there is no self action.

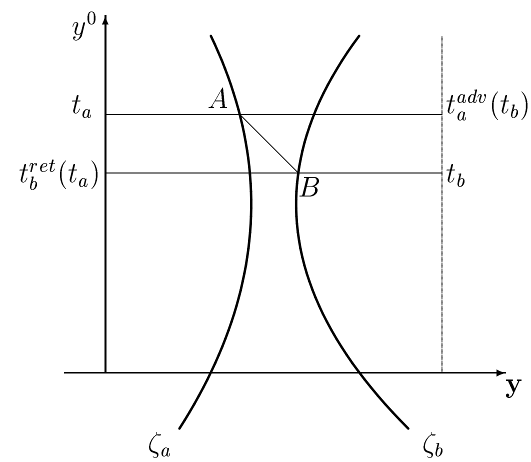

Figure 11: Points $A \in \zeta_a$ and $B \in \zeta_b$ are connectible by a null ray. They are defined by the pair of instants either $(t_a, t_b^{ret}(t_a))$ or $(t_a^{adv}(t_b), t_b)$. Functions $t_b^{ret}(t_a)$ and $t_a^{adv}(t_b)$ are inverses.

The particle a is acted on by the particle b via Lorentz force $F_{ba}^{\alpha} = e_a F_{(b)\beta}^{\alpha} u_a^{\beta}$ where $F_{(b)\beta}^{\alpha}$ is *direct particle field* [14]. By this we mean electromagnetic field generated by b-th particle at point where a-th particle is located. It immediately implies $h = 0$ in expressions (4.6) for the components of electromagnetic fields. Indeed, h is the radius of the circle $S_a \cap S_b$, i.e. of the intersection of spherical fronts of outgoing electromagnetic waves generated by charges (see Figs.4, 5). If we consider the *direct particle field*, the sphere S_a reduces to the point where a-th particle is placed.

To evaluate the retarded field of the 2-nd particle at point $z_1(t_1) \in \zeta_1$ we put $k_1^0 = 0$ and $k_2^0 = q[t_1, t_2^{ret}(t_1)]$ in (4.6). It implies

$$K_2^0 = q, \quad K_2^i = q n_q^i, \quad r_2 = q\left[1 - V_2\right], \quad c_2 = \gamma_2^{-2} + q\dot{V}_2 \tag{C.1}$$

in the expression for $f_{\alpha\beta}^{(2)}$. (It is obvious, that $f_{\alpha\beta}^{(1)}$ vanishes.) All the quantities are evaluated at the moments either t_1 or $t_2^{ret}(t_1)$, $V_2 := (\mathbf{n}_q \cdot \mathbf{v}_2)$ and $\dot{V}_2 := (\mathbf{n}_q \cdot \dot{\mathbf{v}}_2)$.

To find out the advanced field of the 1-st particle at point $z_2(t_2) \in \zeta_2$, we put $k_2^0 = 0$ and $k_1^0 = -q[t_1^{adv}(t_2), t_2]$ in $f_{\alpha\beta}^{(1)}$ given by eq.(4.6). It means

$$K_1^0 = -q, \quad K_1^i = -q n_q^i, \quad r_1 = -q\left[1 - V_1\right], \quad c_1 = \gamma_1^{-2} - q\dot{V}_1 \tag{C.2}$$

where $V_1 := (\mathbf{n}_q \cdot \mathbf{v}_1)$ and $\dot{V}_1 := (\mathbf{n}_q \cdot \dot{\mathbf{v}}_1)$.

In general, to obtain the retarded/advanced field generated by a-th particle at point where b-th particle is located, one should substitute the quantities

$$K_a^0 = \epsilon q, \quad K_a^i = (-1)^a q n_q^i, \quad r_a = \epsilon q \left[1 - (-1)^a \epsilon V_a\right], \quad c_a = \gamma_a^{-2} + (-1)^a q \dot{V}_a \tag{C.3}$$

in eq.(4.6). Parameter ϵ is equal to $+1$ for retarded fields and -1 for advanced ones. Putting eqs.(C.3) in (4.6) we arrive at the following expressions:

$$\begin{aligned}
F_{0i}^{(a)}(\epsilon) &= e_a \left\{ \frac{-\varepsilon_a n_q^i + v_a^i}{q^2 \left(1 - \varepsilon_a V_a\right)^3} \gamma_a^{-2} + (-1)^a \frac{-\varepsilon_a n_q^i + v_a^i}{q \left(1 - \varepsilon_a V_a\right)^3} \dot{V}_a + \epsilon \frac{\dot{v}_a^i}{q \left(1 - \varepsilon_a V_a\right)^2} \right\} \\
F_{ij}^{(a)}(\epsilon) &= e_a \left\{ \varepsilon_a \frac{v_a^i n_q^j - v_a^j n_q^i}{q^2 \left(1 - \varepsilon_a V_a\right)^3} \gamma_a^{-2} + \epsilon \frac{v_a^i n_q^j - v_a^j n_q^i}{q \left(1 - \varepsilon_a V_a\right)^3} \dot{V}_a + (-1)^a \frac{\dot{v}_a^i n_q^j - \dot{v}_a^j n_q^i}{q \left(1 - \varepsilon_a V_a\right)^2} \right\}
\end{aligned} \tag{C.4}$$

where parameters

$$\varepsilon_a = (-1)^a \epsilon. \tag{C.5}$$

The components of Lorentz force a-th charge acting on b-th one are written as follows:

$$\begin{aligned}
\gamma_b^{-1} F_{ab}^0(\epsilon) &= -e_b F_{0i}^{(a)}(\epsilon) v_b^i \qquad \text{(C.6)} \\
&= e_b e_a \left\{ \frac{\varepsilon_a V_b - (\mathbf{v}_a \cdot \mathbf{v}_b)}{q^2 \left(1 - \varepsilon_a V_a\right)^3} \gamma_a^{-2} + (-1)^a \frac{\varepsilon_a V_b - (\mathbf{v}_a \cdot \mathbf{v}_b)}{q \left(1 - \varepsilon_a V_a\right)^3} \dot{V}_a \right. \\
&- \left. \epsilon \frac{(\dot{\mathbf{v}}_a \cdot \mathbf{v}_b)}{q \left(1 - \varepsilon_a V_a\right)^2} \right\} \\
\gamma_b^{-1} F_{ab}^i(\epsilon) &= -e_b F_{0i}^{(a)}(\epsilon) + e_b F_{ij}^{(a)}(\epsilon) v_b^j \qquad \text{(C.7)} \\
&= -e_b e_a \left\{ \varepsilon_a n_q^i \left[\frac{-1 + (\mathbf{v}_a \cdot \mathbf{v}_b)}{q^2 \left(1 - \varepsilon_a V_a\right)^3} \gamma_a^{-2} + (-1)^a \frac{-1 + (\mathbf{v}_a \cdot \mathbf{v}_b)}{q \left(1 - \varepsilon_a V_a\right)^3} \dot{V}_a \right. \right. \\
&+ \left. \epsilon \frac{(\dot{\mathbf{v}}_a \cdot \mathbf{v}_b)}{q \left(1 - \varepsilon_a V_a\right)^2} \right] \\
&+ \frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a} \left[\frac{v_a^i}{q^2 \left(1 - \varepsilon_a V_a\right)^2} \gamma_a^{-2} + (-1)^a \frac{v_a^i}{q \left(1 - \varepsilon_a V_a\right)^2} \dot{V}_a \right. \\
&+ \left. \left. \epsilon \frac{\dot{v}_a^i}{q \left(1 - \varepsilon_a V_a\right)} \right] \right\}.
\end{aligned}$$

All the quantities labelled by a are referred to the instant $t_a^\epsilon(t_b)$ while those supplemented with index b are evaluated at t_b.

D. Difference of work done by "advanced" and retarded Lorenz forces

The retarded, $t_a^{ret}(t_b)$, and "advanced", $t_b^{adv}(t_a)$, instants arise naturally within the integration procedure developed in Section 5 as the end points of "inner" integrals (see eqs.(5.4) and (5.5)). Typical points A (on the worldline of charge a) and B (on the worldline of charge b) interact if the line connecting them is a null ray. It seems, that the interaction

can be both forward (B to A) and backward (A to B) in time (see Fig.11). And yet the retarded causality is not violated. Indeed, we consider the interference of *outgoing* waves present at the observation time t. Both the retarded and "advanced" moments are *before* t.

In this subsection we compare the work done by retarded Lorentz force due to charge b on charge a

$$\int\limits_{-\infty}^{t} dt_a \gamma_a^{-1} F_{ba}^{\mu}[t_a, t_b^{ret}(t_a)] \tag{D.1}$$

and the work done by "advanced" response of charge a on charge b

$$\int\limits_{-\infty}^{t_b^{ret}(t)} dt_b \gamma_b^{-1} F_{ab}^{\mu}[t_a^{adv}(t_b), t_b]. \tag{D.2}$$

The following identity generalises the derivatives of eqs.(3.9), (3.10), (3.14) and (3.15):

$$\frac{dt_a^{\epsilon}(t_b)}{dt_b} = \frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a}. \tag{D.3}$$

Here

$$\varepsilon_a = (-1)^a \epsilon; \tag{D.4}$$

parameter ϵ is equal to $+1$ for retarded instants and -1 for advanced ones. With the help of eq.(D.3) we obtain the following chain of identities:

$$\begin{aligned}
\frac{dq[t_a^{\epsilon}(t_b), t_b]}{dt_b} &= (-1)^a \frac{V_b - V_a}{1 - \varepsilon_a V_a} && \text{(D.5)} \\
\frac{dn_q^i[t_a^{\epsilon}(t_b), t_b]}{dt_b} &= \frac{(-1)^a}{q}\left[v_b^i - v_a^i \frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a} - n_q^i \frac{V_b - V_a}{1 - \varepsilon_a V_a}\right] && \text{(D.6)} \\
\frac{d(\mathbf{n}_q \cdot \mathbf{v}_b)}{dt_b} &= \frac{(-1)^a}{q}\left[-\gamma_b^{-2} - [-1 + (\mathbf{v}_1 \cdot \mathbf{v}_2)]\frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a}\right. \\
&+ \left. \varepsilon_a (V_b - V_a)\frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a}\right] + \dot{V}_b && \text{(D.7)} \\
\frac{d(\mathbf{n}_q \cdot \mathbf{v}_a)}{dt_b} &= \frac{(-1)^a}{q}\left[\gamma_a^{-2}\frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a} - 1 + (\mathbf{v}_1 \cdot \mathbf{v}_2) + \varepsilon_a (V_b - V_a)\right] \\
&+ \dot{V}_a \frac{1 - \varepsilon_a V_b}{1 - \varepsilon_a V_a} && \text{(D.8)}
\end{aligned}$$

To compare (D.1) and (D.2) we change the variables $[t_a^{adv}(t_b), t_b] \mapsto [t_a, t_b^{ret}(t_a)]$ in "advanced" integral:

$$\begin{aligned}
&\int\limits_{-\infty}^{t} dt_a \gamma_a^{-1} F_{ba}^{\mu}[t_a, t_b^{ret}(t_a)] - \int\limits_{-\infty}^{t_b^{ret}(t)} dt_b \gamma_b^{-1} F_{ab}^{\mu}[t_a^{adv}(t_b), t_b] && \text{(D.9)} \\
= &\int\limits_{-\infty}^{t} dt_a \left[\gamma_a^{-1} F_{ba}^{\mu}[t_a, t_b^{ret}(t_a)] - \frac{1 + (-1)^b V_a}{1 + (-1)^b V_b}\gamma_b^{-1} F_{ab}^{\mu}[t_a^{adv}(t_b), t_b]\Big|_{t_b = t_b^{ret}(t_a)}^{t_a^{adv}(t_b) = t_a}\right]
\end{aligned}$$

Using identities (D.6)-(D.8) in the integrand of eq.(D.9), we derive that it is the total time derivative. In other words, the difference of "retarded" work (D.1) and "advanced" one (D.2) is the integral being a function of the end points only.

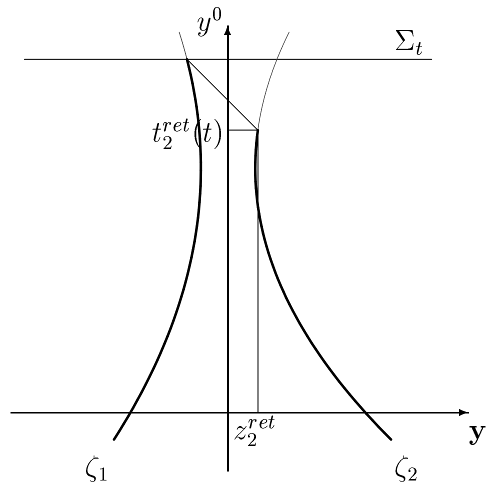

Figure 12: Difference of work done by retarded Lorentz force due to charge 2 on charge 1 and the work done by advanced response of charge 1 is given by eqs.(D.10). All the quantities in the right-hand side of this equation which are labelled by 1 are referred to the instant of observation while those supplemented with index 2 are evaluated at $t_2^{ret}(t)$.

$$\int\limits_{-\infty}^{t} dt_1\gamma_1^{-1}F_{21}^{\mu}[t_1,t_2^{ret}(t_1)] - \int\limits_{-\infty}^{t_2^{ret}(t)} dt_2\gamma_2^{-1}F_{12}^{\mu}[t_1^{adv}(t_2),t_2] \tag{D.10}$$
$$= \left\{ \begin{array}{lcl} \mu=0 & , & -e_1e_2\left[\dfrac{-1+(\mathbf{v}_1\mathbf{v}_2)}{q[1-V_1][1-V_2]} + \dfrac{1}{q[1-V_1]} + \dfrac{1}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t} \\ \mu=i & , & -e_1e_2\left[\dfrac{[-1+(\mathbf{v}_1\mathbf{v}_2)]n_q^i}{q[1-V_1][1-V_2]} + \dfrac{v_1^i}{q[1-V_1]} + \dfrac{v_2^i}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t} \end{array}\right.$$

$$\int\limits_{-\infty}^{t} dt_2\gamma_2^{-1}F_{12}^{\mu}[t_1^{ret}(t_2),t_2] - \int\limits_{-\infty}^{t_1^{ret}(t)} dt_1\gamma_1^{-1}F_{21}^{\mu}[t_1,t_2^{adv}(t_1)] \tag{D.11}$$
$$= \left\{ \begin{array}{lcl} \mu=0 & , & e_1e_2\left[-\dfrac{-1+(\mathbf{v}_1\mathbf{v}_2)}{q[1+V_1][1+V_2]} - \dfrac{1}{q[1+V_1]} - \dfrac{1}{q[1+V_2]}\right]_{t_2\to-\infty}^{t_2=t} \\ \mu=i & , & e_1e_2\left[\dfrac{[-1+(\mathbf{v}_1\mathbf{v}_2)]n_q^i}{q[1+V_1][1+V_2]} - \dfrac{v_1^i}{q[1+V_1]} - \dfrac{v_2^i}{q[1+V_2]}\right]_{t_2\to-\infty}^{t_2=t} \end{array}\right.$$

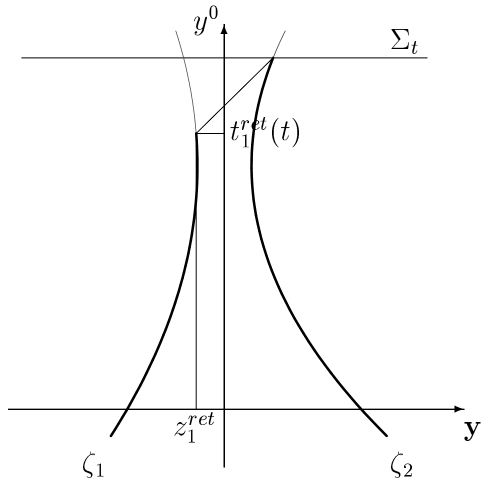

Figure 13: Difference of work done by retarded Lorentz force due to charge 1 on charge 2 and the work done by advanced response of charge 2 is defined by eqs.(D.11). All the quantities in the right-hand side of this equation which are labelled by 2 are referred to the instant of observation while those supplemented with index 1 are evaluated at $t_1^{ret}(t)$.

It is convenient to rewrite the results (D.10) and (D.11) in a manifestly covariant fashion:

$$\int\limits_{-\infty}^{t} dt_a\gamma_a^{-1}F^\mu_{ba}[t_a,t_b^{ret}(t_a)] - \int\limits_{-\infty}^{t_b^{ret}(t)} dt_b\gamma_b^{-1}F^\mu_{ab}[t_a^{adv}(t_b),t_b]$$

$$= \quad (-1)^a e_1e_2\left[\frac{(u_1\cdot u_2)n_q^\mu}{q(n_q\cdot u_1)(n_q\cdot u_2)} - \frac{u_1^\mu}{q(n_q\cdot u_1)} - \frac{u_2^\mu}{q(n_q\cdot u_2)}\right]_{t_a\to-\infty}^{t_a=t} \tag{D.12}$$

Symbols $u_a^\mu, a=1,2$, denotes the (normalized) four-velocity vector $(\gamma_a^{-1},\gamma_a^{-1}v_a^i)$. If the 2-nd particle moves in the retarded field of the 1-st one while the 1-st particle moves in the "advanced" field of the 2-nd one, then $n_q^\mu=(1,n_q^i)$. Four-products of this null vector with four velocities are as follows:

$$(n_q\cdot u_a) = -\gamma_a^{-1}\left[1-V_a\right]. \tag{D.13}$$

If one interchanges the words "first particle" and "second particle" in the above sentences, $n_q^\mu=(-1,n_q^i)$ and we have

$$(n_q\cdot u_a) = \gamma_a^{-1}\left[1+V_a\right]. \tag{D.14}$$

in eq.(D.12).

E. Time integration of $T^{0\mu}_{\rm int}$

In this paper we integrate the interference part of energy-momentum tensor density of two point-like charged particles over three-dimensional hyperplane $\Sigma_t=\{y\in\mathbb{M}_4: y^0=$

$t\}$. An integration hypersurface is a surface of constant value of the obsevation time parameter. Besides t, the set of curvilinear coordinates includes the "individual" retarded times t_1 and t_2, associated with the particles' worldlines, and the angle variable φ. The integration over φ is performed in Appendix A and Appendix B. The crucial issue is that the resulting expressions are the sum of partial derivatives in individual times (see eqs.(5.10) and combination of (5.33) and (5.36)). It allows us to perform the integration over one of the time parameters, either t_1 or t_2. "Retarded" shifts in arguments of particles' individual characteristics such us coordinates, velocities etc. appear on this stage as well as "advanced" ones.

The first double integral involved in the rules either (5.4) or (5.5) defines the integration over "causal" region which is pictured in Figs.6 and 7, while the second one deals with "acausal" region (see Figs.8 and 9). The integration of "causal" type can be handled via the relations (D.3)-(D.8). Their counterparts for "acausal" region look as follows:

$$\begin{aligned}
\frac{dt_1'(t,t_2)}{dt_2} &= -\frac{1-V_2}{1+V_1} &\text{(E.1)}\\
\frac{dq[t_1'(t,t_2),t_2]}{dt_2} &= -\frac{V_1+V_2}{1+V_1} &\text{(E.2)}\\
\frac{dn_q^i[t_1'(t,t_2),t_2]}{dt_2} &= \frac{1}{q}\left[-v_2^i - v_1^i\frac{1-V_2}{1+V_1} + n_q^i\frac{V_1+V_2}{1+V_1}\right] &\text{(E.3)}\\
\frac{d(\mathbf{n}_q\cdot\mathbf{v}_1)}{dt_2} &= \frac{1}{q}\left[\gamma_1^{-2}\frac{1-V_2}{1+V_1} - [1+(\mathbf{v}_1\cdot\mathbf{v}_2)] + V_1 + V_2\right]\\
&- \dot{V}_1\frac{1-V_2}{1+V_1} &\text{(E.4)}\\
\frac{d(\mathbf{n}_q\cdot\mathbf{v}_2)}{dt_2} &= \frac{1}{q}\left[\gamma_2^{-2} - [1+(\mathbf{v}_1\cdot\mathbf{v}_2)]\frac{1-V_2}{1+V_1}\right.\\
&- \left.(V_1+V_2)\frac{1-V_2}{1+V_1}\right] + \dot{V}_2 &\text{(E.5)}
\end{aligned}$$

where $V_a := (\mathbf{n}_q\cdot\mathbf{v}_a)$ and $\dot{V}_a := (\mathbf{n}_q\cdot\dot{\mathbf{v}}_a)$.

The way of integration where all the mixed derivatives are written as $\partial/\partial t_1[\partial G_0/\partial t_2]$ results the expressions placed in the left columnes of Tables. We apply the rules (5.16), (5.20) and (5.24) for the 1-st, 2-nd and 3-rd line, respectively.

If one changes the order of differentiations they obtain the expressions in the right columnes of Tables. For the 1-st and 2-nd lines time integration rules are as follows:

$$\int\limits_{-\infty}^{t_2^{ret}(t)} dt_2\left[G_1 - \frac{1-V_2}{1-V_1}G_2\right]^{t_1=t_1^{adv}(t_2)} \quad \text{(a)}, \int\limits_{-\infty}^{t} dt_2\left[-G_1 + \frac{1+V_2}{1+V_1}G_2\right]_{t_1=t_1^{ret}(t_2)} \quad \text{(b)}. \quad \text{(E.6)}$$

Acausal region is integrated according to the rule (5.24) (3-rd line of the right column).

Table 1. Integral $\int_0^{2\pi}\sqrt{-g}T^{00}_{int}$ has the form of $\partial G_1/\partial t_1 + \partial G_2/\partial t_2 + \partial^2 G_0/\partial t_1\partial t_2$. Integration over time results the expressions in the left column (if mixed derivative is coupled with $\partial G_1/\partial t_1$) or in the right column (if $\partial^2 G_0/\partial t_1\partial t_2$ is added to $\partial G_2/\partial t_2$). Integration over "acausal" region gives the functions of the end points only (see third line).

$\dfrac{\partial}{\partial t_1}\left[\dfrac{\partial G_0}{\partial t_2}\right]$	$\dfrac{\partial}{\partial t_2}\left[\dfrac{\partial G_0}{\partial t_1}\right]$
(5.16) $-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^0_{21}[t_1,t_2^{ret}(t_1)]$ $+e_1e_2\left[\dfrac{1}{2k_2^0}\dfrac{1+V_2}{1-V_2}-\dfrac{1}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t}$	(E.6a) $-\int_{-\infty}^{t_2^{ret}(t)} dt_2\gamma_2^{-1}F^0_{12}[t_1^{adv}(t_2),t_2]$ $+e_1e_2\left[\dfrac{1}{2k_1^0}\dfrac{1+V_1}{1-V_1}+\dfrac{1}{q[1-V_1]}\right]_{t_2\to-\infty}^{t_2\to t_2^{ret}(t)}$
(5.20) $-\int_{-\infty}^{t_1^{ret}(t)} dt_1\gamma_1^{-1}F^0_{21}[t_1,t_2^{adv}(t_1)]$ $+e_1e_2\left[\dfrac{1}{2k_2^0}\dfrac{1-V_2}{1+V_2}+\dfrac{1}{q[1+V_2]}\right]_{t_1\to-\infty}^{t_1\to t_1^{ret}(t)}$	(E.6b) $-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^0_{12}[t_1^{ret}(t_2),t_2]$ $+e_1e_2\left[\dfrac{1}{2k_1^0}\dfrac{1-V_1}{1+V_1}-\dfrac{1}{q[1+V_1]}\right]_{t_2\to-\infty}^{t_2=t}$
(5.24) $-\left.\dfrac{e_1e_2}{2k_2^0}\right\vert_{t_2=t_2^{ret}(t)}^{t_2\to t}$	(5.24) $\left.\dfrac{e_1e_2}{2k_1^0}\right\vert_{t_2\to t_2^{ret}(t)}^{t_2=t}$

Table 2. Integral $\int_0^{2\pi}\sqrt{-g}T^{0i}_{int}$ becomes the combination of partial derivatives in time variables. Structure of this Table is analogous to the structure of Table 1.

$\dfrac{\partial}{\partial t_1}\left[\dfrac{\partial\Lambda}{\partial t_2}\right]$	$\dfrac{\partial}{\partial t_2}\left[\dfrac{\partial\Lambda}{\partial t_1}\right]$
$-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^i_{21}[t_1,t_2^{ret}(t_1)]$ $+e_1e_2\left[\dfrac{n_q^i+v_2^i}{2k_2^0[1-V_2]}-\dfrac{v_2^i}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t}$	$-\int_{-\infty}^{t_2^{ret}(t)} dt_2\gamma_2^{-1}F^i_{12}[t_1^{adv}(t_2),t_2]$ $+e_1e_2\left[\dfrac{n_q^i+v_1^i}{2k_1^0[1-V_1]}+\dfrac{v_1^i}{q[1-V_1]}\right]_{t_2\to-\infty}^{t_2\to t_2^{ret}(t)}$
$-\int_{-\infty}^{t_1^{ret}(t)} dt_1\gamma_1^{-1}F^i_{21}[t_1,t_2^{adv}(t_1)]$ $+e_1e_2\left[\dfrac{-n_q^i+v_2^i}{2k_2^0[1+V_2]}+\dfrac{v_2^i}{q[1+V_2]}\right]_{t_1\to-\infty}^{t_1\to t_1^{ret}(t)}$	$-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^i_{12}[t_1^{ret}(t_2),t_2]$ $+e_1e_2\left[\dfrac{-n_q^i+v_1^i}{2k_1^0[1+V_1]}-\dfrac{v_1^i}{q[1+V_1]}\right]_{t_2\to-\infty}^{t_2=t}$
$e_1e_2\left.\dfrac{n_q^i-v_2^i}{2k_2^0[1-V_2]}\right\vert_{t_2=t_2^{ret}(t)}^{t_2\to t}$	$e_1e_2\left.\dfrac{n_q^i+v_1^i}{2k_1^0[1+V_1]}\right\vert_{t_2\to t_2^{ret}(t)}^{t_2=t}$

Taking into account the relationship (D.12) between work of the "advanced" Lorentz force and the work of the "retarded" one we remove all the "advanced" integrals from these Tables. The final expressions are written in Table 3.

Table 3. The expressions which are placed above double line concern with integration of energy density T^{00}_{int} while ones below double line result from integration of T^{0i}_{int}.

$\frac{\partial}{\partial t_1}\left[\frac{\partial\Lambda}{\partial t_2}\right]$	$\frac{\partial}{\partial t_2}\left[\frac{\partial\Lambda}{\partial t_1}\right]$
$-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^0_{21}[t_1, t_2^{ret}(t_1)]$ $+e_1e_2\left[\frac{1}{2k_2^0}\frac{1+V_2}{1-V_2}-\frac{1}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t}$	$-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^0_{21}[t_1, t_2^{ret}(t_1)]$ $+e_1e_2\left[\frac{1}{2k_1^0}\frac{1+V_1}{1-V_1}-\frac{1}{q[1-V_2]}-\frac{-1+(\mathbf{v}_1\mathbf{v}_2)}{q[1-V_1][1-V_2]}\right]_{t_1\to-\infty}^{t_1\to t}$
$-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^0_{12}[t_1^{ret}(t_2), t_2]$ $+e_1e_2\left[\frac{1}{2k_2^0}\frac{1-V_2}{1+V_2}-\frac{1}{q[1+V_1]}-\frac{-1+(\mathbf{v}_1\mathbf{v}_2)}{q[1+V_1][1+V_2]}\right]_{t_2\to-\infty}^{t_2\to t}$	$-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^0_{12}[t_1^{ret}(t_2), t_2]$ $+e_1e_2\left[\frac{1}{2k_1^0}\frac{1-V_1}{1+V_1}-\frac{1}{q[1+V_1]}\right]_{t_2\to-\infty}^{t_2=t}$
$-\left.\frac{e_1e_2}{2k_2^0}\right\|_{t_2=t_2^{ret}(t)}^{t_2\to t}$	$\left.\frac{e_1e_2}{2k_1^0}\right\|_{t_2\to t_2^{ret}(t)}^{t_2=t}$
$-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^i_{21}[t_1, t_2^{ret}(t_1)]$ $+e_1e_2\left[\frac{n_q^i+v_2^i}{2k_2^0[1-V_2]}-\frac{v_2^i}{q[1-V_2]}\right]_{t_1\to-\infty}^{t_1=t}$	$-\int_{-\infty}^{t} dt_1\gamma_1^{-1}F^i_{21}[t_1, t_2^{ret}(t_1)]$ $+e_1e_2\left[\frac{n_q^i+v_1^i}{2k_1^0[1-V_1]}-\frac{v_2^i}{q[1-V_2]}-\frac{[-1+(\mathbf{v}_1\mathbf{v}_2)]\,n_q^i}{q[1-V_1][1-V_2]}\right]_{t_1\to-\infty}^{t_1\to t}$
$-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^i_{12}[t_1^{ret}(t_2), t_2]$ $+e_1e_2\left[\frac{-n_q^i+v_2^i}{2k_2^0[1+V_2]}-\frac{v_1^i}{q[1+V_1]}+\frac{[-1+(\mathbf{v}_1\mathbf{v}_2)]\,n_q^i}{q[1+V_1][1+V_2]}\right]_{t_2\to-\infty}^{t_2\to t}$	$-\int_{-\infty}^{t} dt_2\gamma_2^{-1}F^i_{12}[t_1^{ret}(t_2), t_2]$ $+e_1e_2\left[\frac{-n_q^i+v_1^i}{2k_1^0[1+V_1]}-\frac{v_1^i}{q[1+V_1]}\right]_{t_2\to-\infty}^{t_2=t}$
$e_1e_2\left.\frac{n_q^i-v_2^i}{2k_2^0[1-V_2]}\right\|_{t_2=t_2^{ret}(t)}^{t_2\to t}$	$e_1e_2\left.\frac{n_q^i+v_1^i}{2k_1^0[1+V_1]}\right\|_{t_2\to t_2^{ret}(t)}^{t_2=t}$

References

[1] Dirac P A M 1938 *Proc.Roy.Soc.* **A167** 148

[2] López C A and Villarroel D 1975 *Phys.Rew.D* **11** 2724

[3] Yaremko Yu 2002 *J. Phys.A:Math.Gen.* **35** 831; 2003 **36** 5159

[4] Yaremko Yu 2002 *J. Phys.A:Math.Gen.* **35** 9441

[5] Yaremko Yu 2003 *J. Phys.A:Math.Gen.* **36** 5149

[6] Yaremko Yu 2004 *J. Phys.A:Math.Gen.* **37** 1079

[7] Rohrlich F 1990 *Classical Charged Particles* (Redwood City, CA: Addison-Wesley)

[8] Gaida R P, Klyuchkovsky Yu B and Tretyak V I 1983 *Theor.Math.Phys.* **55** 372

[9] Poisson E *An introduction to the Lorentz-Dirac equation*, Prepr. gr-qc/9912045 10 Dec 1999.

[10] Poisson E *The motion of point particles in curved spacetime*, Prepr. gr-qc/0306052 15 Jul 2003.

[11] Teitelboim C 1970 *Phys.Rev.D* **1** 1572

[12] Wheeler J A and Feynman R P 1945 *Rev.Mod.Phys.* **17** 157

[13] Kolář I, Michor P W and Slovák J 1993 *Natural Operations in Differential Geometry* (Springer-Verlag New York Berlin Heidelberg)

[14] Hoyle F and Narlikar J V 1995 *Rev.Mod.Phys.* **67** 113